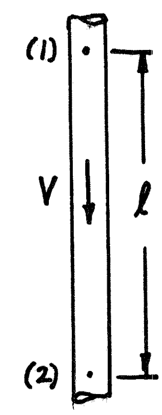
8.83

8.83 Water flows downward through a vertical smooth pipe. When the flowrate is 0.5 ft^3/s there is no change in pressure along the pipe. Determine the diameter of the pipe.



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + f\frac{\ell}{D}\frac{V^2}{2g}$$

where $p_1 = p_2$, $V_1 = V_2 = V$, and $z_1 - z_2 = \ell$

Thus,

$$\ell = f\frac{\ell}{D}\frac{V^2}{2g}, \text{ or } 1 = \frac{f}{D}\frac{V^2}{2g} \qquad (1)$$

Also,

$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}$ so that Eq. (1) becomes $1 = \frac{f}{D}\frac{\left(\frac{4Q}{\pi D^2}\right)^2}{2g}$

or

$$D^5 = \frac{8}{\pi^2} f \frac{Q^2}{g} = \frac{8}{\pi^2} f \frac{(0.5)^2}{32.2} \quad \text{or} \quad D = 0.363\, f^{1/5} \qquad (2)$$

Also,

$$Re = \frac{\rho V D}{\mu} = \frac{1.94\left(\frac{4Q}{\pi D^2}\right)D}{\mu} = \frac{1.94\left(\frac{4(0.5)}{\pi}\right)}{2.34\times 10^{-5}\, D} \quad \text{or} \quad Re = \frac{5.28\times 10^4}{D} \qquad (3)$$

From Fig. 8.20 with $\frac{\varepsilon}{D} = 0$ we have $f = f(Re, \frac{\varepsilon}{D} = 0)$

Trial and error solution: 3 unknowns (D, Re, f) and 3 equations ((2), (3), and Fig. 8.20)

Assume $f = 0.02$ so from Eq. (2), $D = 0.166$ ft and from Eq. (3), $Re = 3.18 \times 10^5$. Thus, from Fig. 8.20, $f = 0.014 \neq 0.02$

Assume $f = 0.014$ so that $D = 0.155$ ft and $Re = 3.42 \times 10^4$

Thus, from Fig. 8.20, $f \approx 0.14$ which checks with the assumed value.

Thus, $D = \underline{\underline{0.155 \text{ ft}}}$

An alternative method is to use the Colebrook equation, Eq. 8.35, with $\varepsilon/D = 0$, rather than the Moody chart, Fig. 8.20. Thus, $\frac{1}{\sqrt{f}} = -2.0 \log\left(2.51/Re\sqrt{f}\right)$ which combined with Eqs. (2) and (3) gives

$$1/(D/0.363)^{5/2} = -2.0 \log\left[2.51 D/(5.28\times 10^4 (D/0.363)^{5/2})\right] \qquad (4)$$

Using a computer root-finding program gives the solution of Eq. (4) as $D = 0.155$ ft as obtaing by the above trial and error method.