8.82

8.82 Determine the diameter of a steel pipe that is to carry 2,000 gal/min of gasoline with a pressure drop of 5 psi per 100 ft of horizontal pipe.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g}, \text{ where } z_1 = z_2 \text{ and } V_1 = V_2$$

Thus,

$$p_1 - p_2 = f\frac{\ell}{D}\frac{1}{2}\rho V^2 \text{ with } p_1 - p_2 = 5\frac{lb}{in.^2}\left(144\frac{in.^2}{ft^2}\right),\ \ell = 100\,ft, \qquad (1)$$

$$\rho = 1.32\frac{slugs}{ft^3},\ \mu = 6.5\times10^{-6}\frac{lb\cdot s}{ft^2}, \text{ and}$$

$$V = \frac{Q}{A} = \frac{\left(2000\frac{gal}{min}\right)\left(\frac{1\,min}{60\,s}\right)\left(231\frac{in.^3}{gal}\right)\left(\frac{1}{1728}\frac{ft^3}{in.^3}\right)}{\frac{\pi}{4}D^2}, \text{ or } V = \frac{5.67}{D^2}\frac{ft}{s} \text{ with } D \sim ft$$

Hence, Eq. (1) gives:

$$5(144)\frac{lb}{ft^2} = f\left(\frac{100\,ft}{D\,ft}\right)\frac{1}{2}\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)^2$$

or

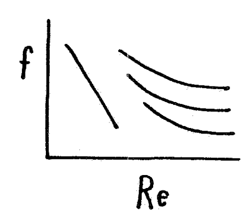
$$D = 1.24\, f^{1/5} \qquad (2)$$

Also,
$$Re = \frac{\rho V D}{\mu} = \frac{\left(1.32\frac{slugs}{ft^3}\right)\left(\frac{5.67}{D^2}\frac{ft}{s}\right)D\,ft}{6.5\times10^{-6}\frac{lb\cdot s}{ft^2}}, \text{ or } Re = \frac{1.15\times10^6}{D} \qquad (3)$$

and

$$\frac{\varepsilon}{D} = \frac{0.00015}{D}, \text{ where } D \sim ft \qquad (4)$$

Finally, the fourth equation is the Moody chart (or the Colebrook equation) (5)



Note: 4 equations ((2), (3), (4), and (5)) and 4 unknowns (f, $\frac{\varepsilon}{D}$, D, Re)

Trial and error solution:

Guess $f = 0.02 \xrightarrow{(2)} D = 0.567\,ft$ $\xrightarrow{(3)} Re = 2.03\times10^6$, $\xrightarrow{(4)} \frac{\varepsilon}{D} = 0.000265$ $\xrightarrow{(5)} f = 0.0148 \neq 0.02$

Thus, the guessed value is not correct.

Guess $f = 0.0148 \xrightarrow{(2)} D = 0.534\,ft$ $\xrightarrow{(3)} Re = 2.15\times10^6$, $\xrightarrow{(4)} \frac{\varepsilon}{D} = 0.000281$ $\xrightarrow{(5)} f = 0.0150 \approx 0.0148$

Thus, $D = 1.24(0.0150)^{1/5} = \underline{\underline{0.535\,ft}}$

By using the Colebrook equation, Eq. 8.35, rather than the Moody chart, Eq. (5), we have

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right] \text{ which, using Eqs (2), (3), and (4) is,}$$

$$\frac{1}{(D/1.24)^{5/2}} = -2.0\log\left[\frac{0.00015}{3.7\,D} + \frac{2.51\,D}{1.15\times10^6(D/1.24)^{5/2}}\right]$$

Using a computer root-finding program to solve Eq. (6) gives $D = 0.536\,ft$ which is consistent with the above trial and error solution.