7.22

**7.22** The height, $h$, that a liquid will rise in a capillary tube is a function of the tube diameter, $D$, the specific weight of the liquid, $\gamma$, and the surface tension, $\sigma$. Perform a dimensional analysis using both the *FLT* and *MLT* systems for basic dimensions. Note: The results should obviously be the same regardless of the system of dimensions used. If your analysis indicates otherwise, go back and check your work giving particular attention to the required number of reference dimensions.

$$h = f(D, \gamma, \sigma)$$

Using FLT system:

$h \doteq L \qquad D \doteq L \qquad \gamma \doteq FL^{-3} \qquad \sigma \doteq FL^{-1}$

From the pi theorem, $4-2=2$ pi terms required.

By inspection, for $\Pi_1$ (containing $h$):

$$\Pi_1 = \frac{h}{D}$$

which is obviously dimensionless.

For $\Pi_2$ (containing $\gamma$ and $\sigma$):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} \doteq \frac{FL^{-1}}{(FL^{-3})(L)^2} = F^0L^0$$

Thus,

$$\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)$$

Using MLT system:

$h \doteq L \qquad D \doteq L \qquad \gamma \doteq ML^{-2}T^{-2} \qquad \sigma = MT^{-2}$

Although there appears to be 3 reference dimensions, only 2 reference dimensions are actually required ($L$ and $MT^{-2}$) to describe the variables. By inspection, for $\Pi_1$ (see above)

$$\Pi_1 = \frac{h}{D}$$

and for $\Pi_2$ (containing $\gamma$ and $\sigma$):

$$\Pi_2 = \frac{\sigma}{\gamma D^2} = \frac{MT^{-2}}{(ML^{-2}T^{-2})(L)^2} = M^0L^0T^0$$

Thus, (as above)

$$\frac{h}{D} = \phi\left(\frac{\sigma}{\gamma D^2}\right)$$