7.20

7.20 A cylinder with a diameter, D, floats upright in a liquid as shown in Fig. P7.20. When the cylinder is displaced slightly along its vertical axis it will oscillate about its equilibrium position with a frequency, ω. Assume that this frequency is a function of the diameter, D, the mass of the cylinder, m, and the specific weight, γ, of the liquid. Determine, with the aid of dimensional analysis, how the frequency is related to these variables. If the mass of the cylinder were increased, would the frequency increase or decrease?

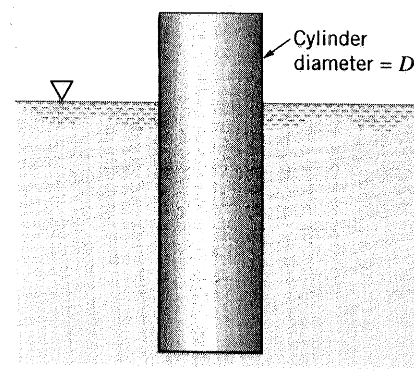

FIGURE P7.20

$$\omega = f(D, m, \gamma)$$

$\omega \doteq T^{-1}$ $\quad D \doteq L$ $\quad m \doteq FL^{-1}T^2$ $\quad \gamma \doteq FL^{-3}$

From the pi theorem, $4-3 = 1$ pi term required.

By inspection:

$$\Pi_1 = \frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{FL^{-1}T^2}{FL^{-3}}} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} \doteq \frac{(T^{-1})}{(L)}\sqrt{\frac{M}{ML^{-2}T^{-2}}} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

Since there is only 1 pi term, it follows that

$$\frac{\omega}{D}\sqrt{\frac{m}{\gamma}} = C$$

where C is a constant. Thus,

$$\underline{\underline{\omega = CD\sqrt{\frac{\gamma}{m}}}}$$

From this result it follows that if m is increased ω will <u>decrease</u>.