7.19

7.19 One type of viscometer consists of an open reservoir with a small diameter tube at the bottom as illustrated in Fig. P7.19. To measure viscosity the system is filled with the liquid of interest and the time required for the liquid level to fall from level H_i to H_f is determined. Use dimensional analysis to obtain a relationship between the viscosity, μ, and the draining time, τ. Assume that the other variables involved are the initial head, H_i, the final head, H_f, the tube diameter, D, and the specific weight of the liquid, γ.

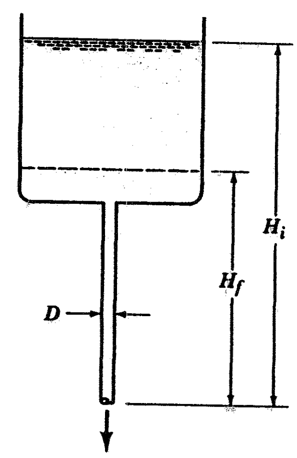

FIGURE P7.19

$$\tau = f(D, H_i, H_f, \mu, \gamma)$$

$\tau \doteq T \quad D \doteq L \quad H_i \doteq L \quad H_f \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3}$

From the pi theorem, $6-3=3$ pi terms required.
By inspection, for Π_1 (containing τ):

$$\Pi_1 = \frac{\tau\gamma D}{\mu} \doteq \frac{(T)(FL^{-3})(L)}{FL^{-2}T} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\tau\gamma D}{\mu} \doteq \frac{(T)(ML^{-2}T^{-2})(L)}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \quad \therefore \text{ OK}$$

For Π_2 (containing H_i):

$$\Pi_2 = \frac{H_i}{D}$$

which is obviously dimensionless. Similarly,

$$\Pi_3 = \frac{H_f}{D}$$

Thus,

$$\frac{\tau\gamma D}{\mu} = \phi\left(\frac{H_i}{D}, \frac{H_f}{D}\right)$$

and for a fixed geometry (including H_i and H_f)

$$\frac{\tau\gamma D}{\mu} = K \qquad (1)$$

where K is a constant, depending on H_i/D and H_f/D.
From Eq. (1)

$$\mu = \frac{\gamma D}{K}\tau$$

so that

$$\underline{\underline{\mu = K_1 \gamma \tau}}$$

where $K_1 = D/K$ and K_1 is a constant for a fixed geometry.