7.18

7.18 The pressure drop, Δp, along a straight pipe of diameter D has been experimentally studied, and it is observed that for laminar flow of a given fluid and pipe, the pressure drop varies directly with the distance, ℓ, between pressure taps. Assume that Δp is a function of D and ℓ, the velocity, V, and the fluid viscosity, μ. Use dimensional analysis to deduce how the pressure drop varies with pipe diameter.

$$\Delta p = f(D, \ell, V, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D \doteq L$ $\quad \ell \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $5-3=2$ pi terms required.

By inspection, for Π_1 (containing Δp):

$$\Pi_1 = \frac{\Delta p\, D}{\mu V} \doteq \frac{(FL^{-2})(L)}{(FL^{-2}T)(LT^{-1})} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\Delta p\, D}{\mu V} \doteq \frac{(ML^{-1}T^{-2})(L)}{(ML^{-1}T^{-1})(LT^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2 (containing ℓ):

$$\Pi_2 = \frac{\ell}{D}$$

which is obviously dimensionless. Thus,

$$\frac{\Delta p\, D}{\mu V} = \phi\left(\frac{\ell}{D}\right) \qquad (1)$$

From the statement of the problem, $\Delta p \propto \ell$ so that Eq. (1) must be of the form

$$\frac{\Delta p\, D}{\mu V} = K\frac{\ell}{D}$$

where K is some constant. It thus follows that

$$\Delta p \propto \frac{1}{D^2}$$

for a given velocity.