7.16

7.16 Assume that the drag, $\mathcal{D}$, on an aircraft flying at supersonic speeds is a function of its velocity, V, fluid density, ρ, speed of sound, c, and a series of lengths, $\ell_1, \ldots, \ell_i$, which describe the geometry of the aircraft. Develop a set of pi terms that could be used to investigate experimentally how the drag is affected by the various factors listed. Form the pi terms by inspection.

$$\mathcal{D} = f(V, \rho, c, \ell_1, \ldots \ell_i)$$

$\mathcal{D} \doteq F \quad V = LT^{-1} \quad \rho \doteq FL^{-4}T^2 \quad c \doteq LT^{-1}$ all lengths, $\ell_i \doteq L$

From the pi theorem, $(4+i)-3 = 1+i$ pi terms required, where i is the number of length terms ($i = 1, 2, 3$, etc.).

By inspection, for Π_1 (containing $\mathcal{D}$):

$$\Pi_1 = \frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{F}{(FL^{-4}T^2)(LT^{-1})^2(L)^2} \doteq F^0L^0T^0$$

Check using MLT:

$$\frac{\mathcal{D}}{\rho V^2 \ell_1^2} \doteq \frac{MLT^{-2}}{(ML^{-3})(LT^{-1})^2(L)^2} \doteq M^0L^0T^0 \quad \therefore \text{OK}$$

For Π_2 (containing c):

$$\Pi_2 = \frac{c}{V} \quad \text{or} \quad \frac{V}{c}$$

and both are obviously dimensionless.

For all other pi terms containing ℓ_i

$$\Pi_i = \frac{\ell_i}{\ell_1}$$

and these terms involving the ℓ's are obviously dimensionless.

Thus,

$$\underline{\underline{\frac{\mathcal{D}}{\rho V^2 \ell_1^2} = \phi\left(\frac{V}{c}, \frac{\ell_i}{\ell_1}\right)}}$$

where $\frac{\ell_i}{\ell_1}$ is a series of pi terms, $\frac{\ell_2}{\ell_1}, \frac{\ell_3}{\ell_1}$, etc.