7.12

7.12 The velocity, V, of a spherical particle falling slowly in a viscous liquid can be expressed as

$$V = f(d, \mu, \gamma, \gamma_s)$$

where d is the particle diameter, μ the liquid viscosity, and γ and γ_s the specific weight of the liquid and particle, respectively. Develop a set of dimensionless parameters that can be used to investigate this problem.

$$V \doteq LT^{-1} \quad d \doteq L \quad \mu \doteq FL^{-2}T \quad \gamma \doteq FL^{-3} \quad \gamma_s \doteq FL^{-3}$$

From the pi theorem, $5-3 = 2$ pi terms required. Use d, μ, and γ as repeating variables. Thus,

$$\Pi_1 = V d^a \mu^b \gamma^c$$

and

$$(LT^{-1})(L)(FL^{-2}T)^b (FL^{-3})^c \doteq F^0 L^0 T^0$$

so that

$$b + c = 0 \qquad (\text{for } F)$$

$$1 + a - 2b - 3c = 0 \qquad (\text{for } L)$$

$$-1 + b = 0 \qquad (\text{for } T)$$

It follows that $a = -2$, $b = 1$, $c = -1$, and therefore

$$\Pi_1 = \frac{V\mu}{d^2\gamma}$$

Check dimensions using MLT system:

$$\frac{V\mu}{d^2\gamma} \doteq \frac{(LT^{-1})(ML^{-1}T^{-1})}{(L)^2(ML^{-2}T^{-2})} \doteq M^0L^0T^0 \quad \therefore \text{ok}$$

For Π_2:

$$\Pi_2 = \gamma_s d^a \mu^b \gamma^c$$

$$(FL^{-3})(L)^a(FL^{-2}T)^b(FL^{-3})^c \doteq F^0L^0T^0$$

$$1 + b + c = 0 \qquad (\text{for } F)$$

$$-3 + a - 2b - 3c = 0 \qquad (\text{for } L)$$

$$b = 0 \qquad (\text{for } T)$$

It follows that $a = 0$, $b = 0$, $c = -1$, and therefore

$$\Pi_2 = \frac{\gamma_s}{\gamma}$$

which is obviously dimensionless.

Thus,

$$\underline{\underline{\frac{V\mu}{d^2\gamma} = \phi\left(\frac{\gamma_s}{\gamma}\right)}}$$