7.7

7.7 When a small pebble is dropped into a liquid, small waves travel outward as shown in Fig. P7.7. The speed of these waves, c, is assumed to be a function of the liquid density, ρ, the wavelength, λ, the wave height, h, and the surface tension of the liquid, σ. Use h, ρ, and σ as repeating variables to determine a suitable set of pi terms that could be used to describe this problem.

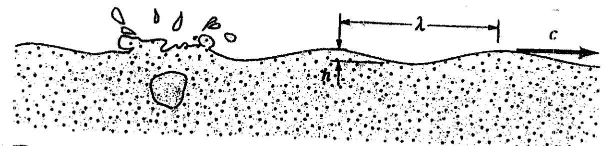

FIGURE P7.7

$$c = f(\rho, \lambda, h, \sigma)$$

$c \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad \lambda \doteq L$ $\quad h \doteq L$ $\quad \sigma \doteq FL^{-1}$

From the pi theorem, 5 − 3 = 2 pi terms required. Use h, ρ, and σ as repeating variables. Thus,

$$\Pi_1 = c\, h^a \rho^b \sigma^c$$

and

$$(LT^{-1})(L)^a(FL^{-4}T^{2})^b(FL^{-1})^c \doteq F^0L^0T^0$$

so that

$$b + c = 0 \qquad (\text{for } F)$$
$$1 + a - 4b - c = 0 \qquad (\text{for } L)$$
$$-1 + 2b = 0 \qquad (\text{for } T)$$

It follows that $a = 1/2$, $b = 1/2$, $c = -1/2$, and therefore

$$\Pi_1 = c\, h^{1/2}\rho^{1/2}\sigma^{-1/2} = c\sqrt{\frac{\rho h}{\sigma}}$$

Check dimensions:

$$c\sqrt{\frac{\rho h}{\sigma}} \doteq (LT^{-1})\left[\frac{(FL^{-4}T^{2})(L)}{(FL^{-1})}\right]^{1/2} \doteq F^0L^0T^0 \therefore \text{OK}$$

For Π_2:

$$\Pi_2 = \lambda\, h^a \rho^b \sigma^c$$

and

$$(L)(L)^a(FL^{-4}T^{2})^b(FL^{-1})^c \doteq F^0L^0T^0$$

so that

$$b + c = 0 \qquad (\text{for } F)$$
$$1 + a - 4b - c = 0 \qquad (\text{for } L)$$
$$2b = 0 \qquad (\text{for } T)$$

It follows that $a = -1$, $b = 0$, $c = 0$, so that

$$\Pi_2 = \frac{\lambda}{h}$$

which is obviously dimensionless. Thus,

$$c\sqrt{\frac{\rho h}{\sigma}} = \phi\left(\frac{\lambda}{h}\right)$$