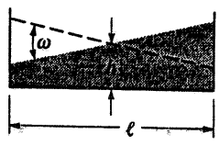
7.6

7.6 Water sloshes back and forth in a tank as shown in Fig. P7.6. The frequency of sloshing, ω, is assumed to be a function of the acceleration of gravity, g, the average depth of the water, h, and the length of the tank, ℓ. Develop a suitable set of dimensionless parameters for this problem using g and ℓ as repeating variables.



■ FIGURE P7.6

$$\omega = f(g, h, \ell)$$

$\omega \doteq T^{-1}$ $\quad g \doteq LT^{-2}$ $\quad h \doteq L$ $\quad \ell \doteq L$

From the pi theorem, $4 - 2 = 2$ dimensionless parameters required. Use g and ℓ as repeating variables, Thus,

$$\pi_1 = \omega g^a \ell^b$$

and

$$(T^{-1})(LT^{-2})^a (L)^b \doteq L^0 T^0$$

so that

$$a + b = 0 \qquad \text{(for } L\text{)}$$
$$-1 - 2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = -1/2$, $b = 1/2$, and therefore

$$\pi_1 = \omega \sqrt{\frac{\ell}{g}}$$

Check dimensions:

$$\omega \sqrt{\frac{\ell}{g}} \doteq \frac{1}{T} \sqrt{\frac{L}{LT^{-2}}} \doteq L^0 T^0 \quad \therefore \text{ OK}$$

For π_2:

$$\pi_2 = h g^a \ell^b$$

$$L (LT^{-2})^a (L)^b \doteq L^0 T^0$$

$$1 + a + b = 0 \qquad \text{(for } L\text{)}$$
$$-2a = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 0$, $b = -1$, and therefore

$$\pi_2 = \frac{h}{\ell}$$

and π_2 is obviously dimensionless. Thus,

$$\omega \sqrt{\frac{\ell}{g}} = \phi\left(\frac{h}{\ell}\right)$$