7.5

7.5 At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity, V, in the larger pipe, and the fluid density, ρ, and viscosity, μ. Use D_1, V, and μ as repeating variables to determine a suitable set of dimensionless parameters. Why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

$$\Delta p = f(D_1, D_2, V, \rho, \mu)$$

$\Delta p \doteq FL^{-2}$ $\quad D_1 \doteq L$ $\quad D_2 \doteq L$ $\quad V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^2$ $\quad \mu \doteq FL^{-2}T$

From the pi theorem, $6-3=3$ dimensionless parameters required. Use D_1, V, and μ as repeating variables. Thus,

$$\Pi_1 = \Delta p\, D_1^a V^b \mu^c$$

and

$$(FL^{-2})(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

so that

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-2 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=1$, $b=-1$, $c=-1$, and therefore

$$\Pi_1 = \frac{\Delta p\, D_1}{V\mu}$$

Check dimensions using MLT system:

$$\frac{\Delta p\, D_1}{V\mu} \doteq \frac{(ML^{-1}T^{-2})(L)}{(LT^{-1})(ML^{-1}T^{-1})} \doteq M^0L^0T^0 \quad \therefore \text{ ok}$$

For Π_2:

$$\Pi_2 = D_2\, D_1^a V^b \mu^c$$

$$L\,(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$c = 0 \qquad \text{(for } F\text{)}$$

$$1 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$-b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a=-1$, $b=0$, $c=0$, and therefore

$$\Pi_2 = \frac{D_2}{D_1}$$

(con't)

7.5 (Con't)

Π_2 is obviously dimensionless.

For Π_3:

$$\Pi_3 = \rho D_1^a V^b \mu^c$$

$$(FL^{-4}T^2)(L)^a(LT^{-1})^b(FL^{-2}T)^c \doteq F^0L^0T^0$$

$$1 + c = 0 \qquad \text{(for } F\text{)}$$

$$-4 + a + b - 2c = 0 \qquad \text{(for } L\text{)}$$

$$2 - b + c = 0 \qquad \text{(for } T\text{)}$$

It follows that $a = 1,\ b = 1,\ c = -1$ and therefore

$$\Pi_3 = \frac{\rho D_1 V}{\mu}$$

Check dimensions using MLT system:

$$\frac{\rho D_1 V}{\mu} \doteq \frac{(ML^{-3})(L)(LT^{-1})}{ML^{-1}T^{-1}} \doteq M^0L^0T^0 \qquad \therefore \text{OK}$$

Thus,

$$\underline{\underline{\frac{\Delta p\, D_1}{V\mu} = \phi\left(\frac{D_2}{D_1}, \frac{\rho D_1 V}{\mu}\right)}}$$

From the continuity equation,

$$V \frac{\pi}{4} D_1^2 = V_s \frac{\pi}{4} D_2^2$$

where V_s is the velocity in the smaller pipe. Since

$$V_s = \left(\frac{D_1}{D_2}\right)^2 V$$

V_s is not independent of D_1, D_2, and V and therefore should <u>not</u> be included as an independent variable.