6.92

6.92 **(a)** Show that for Poiseuille flow in a tube of radius R the magnitude of the wall shearing stress, τ_{rz}, can be obtained from the relationship

$$|(\tau_{rz})_{\text{wall}}| = \frac{4\mu Q}{\pi R^3}$$

for a Newtonian fluid of viscosity μ. The volume rate of flow is Q. **(b)** Determine the magnitude of the wall shearing stress for a fluid having a viscosity of 0.004 N·s/m² flowing with an average velocity of 130 mm/s in a 2-mm-diameter tube.

(a) $$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad \text{(Eq. 6.126f)}$$

For Poiseuille flow in a tube, $v_r = 0$, and therefore

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Since,

$$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{(Eq. 6.154)}$$

and $v_{max} = 2V$, where V is the mean velocity, it follows that

$$\frac{\partial v_z}{\partial r} = -\frac{4Vr}{R^2}$$

Thus, at the wall $(r=R)$,

$$(\tau_{rz})_{wall} = -\frac{4\mu V}{R}$$

and with $Q = \pi R^2 V$

$$\left|(\tau_{rz})_{wall}\right| = \frac{4\mu Q}{\pi R^3}$$

(b) $$\left|(\tau_{rz})_{wall}\right| = \frac{4\mu V}{R} = \frac{4\left(0.004\ \frac{N\cdot s}{m^2}\right)\left(0.130\ \frac{m}{s}\right)}{\left(\frac{0.002}{2}\ m\right)}$$

$$= 2.08\ Pa$$