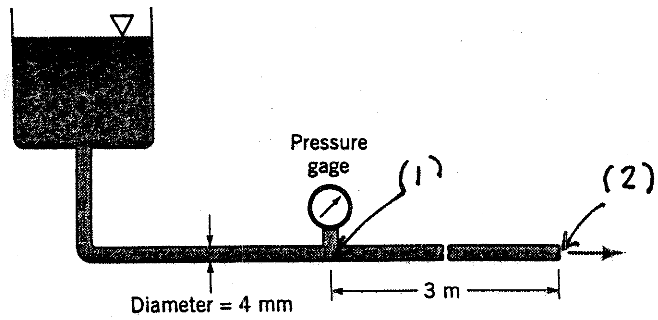
6.90

6.90 A simple flow system to be used for steady flow tests consists of a constant head tank connected to a length of 4-mm-diameter tubing as shown in Fig. P6.90. The liquid has a viscosity of 0.015 N · s/m², a density of 1200 kg/m³, and discharges into the atmosphere with a mean velocity of 2 m/s. **(a)** Verify that the flow will be laminar. **(b)** The flow is fully developed in the last 3 m of the tube. What is the pressure at the pressure gage? **(c)** What is the magnitude of the wall shearing stress, τ_{rz}, in the fully developed region?



■ FIGURE P6.90

(a) Check Reynolds number to determine if flow is laminar:

$$Re = \frac{\rho V(2R)}{\mu} = \frac{(1200\ \frac{kg}{m^3})(2\ \frac{m}{s})(0.004\,m)}{0.015\ \frac{N\cdot s}{m^2}} = 640$$

Since the Reynolds number is well below 2100 the flow is laminar.

(b) For laminar flow,

$$V = \frac{R^2}{8\mu}\frac{\Delta p}{\ell} \qquad (\text{Eq. } 6.152)$$

Since $\Delta p = p_1 - p_2 = p_1 - 0$ (see figure)

$$p_1 = \frac{8\mu V \ell}{R^2} = \frac{8(0.015\ \frac{N\cdot s}{m^2})(2\ \frac{m}{s})(3\,m)}{(\frac{0.004}{2}\,m)^2} = \underline{\underline{180\ kPa}}$$

(c)

$$\tau_{rz} = \mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) \qquad (\text{Eq. } 6.126f)$$

For fully developed pipe flow, $v_r = 0$, so that

$$\tau_{rz} = \mu \frac{\partial v_z}{\partial r}$$

Also,

$$v_z = v_{max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad (\text{Eq. } 6.154)$$

and with $v_{max} = 2V$, where V is the mean velocity

$$\tau_{rz} = 2V\mu\left(-\frac{2r}{R^2}\right)$$

Thus, at the wall, $r = R$,

$$\left|(\tau_{rz})_{wall}\right| = \left|-\frac{4V\mu}{R}\right| = \left|-\frac{4(2\ \frac{m}{s})(0.015\ \frac{N\cdot s}{m^2})}{(\frac{0.004}{2}\,m)}\right| = \underline{\underline{60.0\ \frac{N}{m^2}}}$$