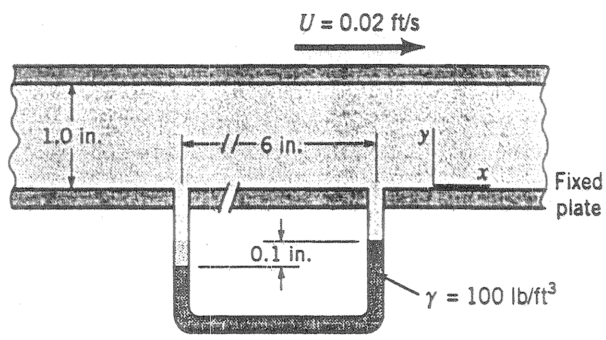
6.83

6.83 A viscous fluid (specific weight = 80 lb/ft^3; viscosity = 0.03 lb · s/ft^2) is contained between two infinite, horizontal parallel plates as shown in Fig. P6.83. The fluid moves between the plates under the action of a pressure gradient, and the upper plate moves with a velocity U while the bottom plate is fixed. A U-tube manometer connected between two points along the bottom indicates a differential reading of 0.1 in. If the upper plate moves with a velocity of 0.02 ft/s, at what distance from the bottom plate does the maximum velocity in the gap between the two plates occur? Assume laminar flow.



■ FIGURE P6.83

$$u = U\frac{y}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - by) \qquad \text{(Eq. 6.140)}$$

Maximum velocity will occur at distance y_m where $\frac{du}{dy} = 0$.

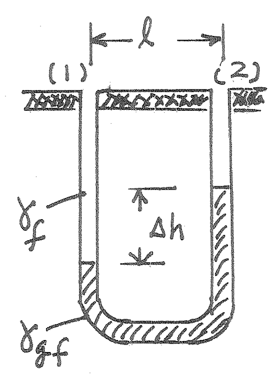
Thus,

$$\frac{du}{dy} = \frac{U}{b} + \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(2y - b)$$

and for $\frac{du}{dy} = 0$

$$y_m = -\frac{\mu U}{b\left(\frac{\partial p}{\partial x}\right)} + \frac{b}{2} \qquad (1)$$

For manometer (see figure to right),



$$p_1 + \gamma_f \Delta h - \gamma_{gf} \Delta h = p_2$$

or

$$p_1 - p_2 = (\gamma_{gf} - \gamma_f)\Delta h$$

$$= \left(100\,\frac{lb}{ft^3} - 80\,\frac{lb}{ft^3}\right)\left(\frac{0.1\text{ in.}}{12\,\frac{in.}{ft}}\right) = 0.167\,\frac{lb}{ft^2}$$

Also,

$$-\frac{\partial p}{\partial x} = \frac{p_1 - p_2}{\ell} = \frac{0.167\,\frac{lb}{ft^2}}{\left(\frac{6\text{ in.}}{12\,\frac{in.}{ft}}\right)} = 0.334\,\frac{lb}{ft^3}$$

Thus, from Eq. (1)

$$y_m = -\frac{\left(0.03\,\frac{lb\cdot s}{ft^2}\right)\left(0.02\,\frac{ft}{s}\right)}{\left(\frac{1.0\text{ in.}}{12\,\frac{in.}{ft}}\right)\left(-0.334\,\frac{lb}{ft^3}\right)} + \frac{\frac{1.0\text{ in.}}{12\,\frac{in.}{ft}}}{2}$$

$$= 0.0632\text{ ft}\left(\frac{12\text{ in.}}{ft}\right) = \underline{\underline{0.759\text{ in.}}}$$