

6.80

6.80 An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in Fig. P6.80. The two plates move in opposite directions with constant velocities, U_1 and U_2 , as shown. The pressure gradient in the x direction is zero and the only body force is due to the fluid weight. Use the Navier-Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

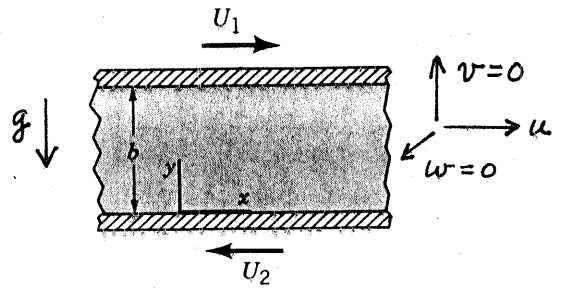


FIGURE P6.80

For the specified conditions, $v=0$, $w=0$, $\frac{\partial P}{\partial x}=0$, and $g_x=0$, so that the x -component of the Navier-Stokes equations (Eq. 6.127a) reduces to

$$\frac{d^2 u}{dy^2} = 0 \quad (1)$$

Integration of Eq. (1) yields

$$u = C_1 y + C_2 \quad (2)$$

For $y=0$, $u = -U_2$ and therefore from Eq. (2)

$$C_2 = -U_2$$

For $y=b$, $u = U_1$, so that

$$U_1 = C_1 b - U_2$$

or

$$C_1 = \frac{U_1 + U_2}{b}$$

Thus,

$$\underline{u = \left(\frac{U_1 + U_2}{b} \right) y - U_2}$$