6.78

6.78 A fluid of density ρ flows steadily *downward* between the two vertical infinite, parallel plates shown in the figure for Problem 6.77. The flow is fully developed and laminar. Make use of the Navier–Stokes equation to determine the relationship between the discharge and the other parameters involved, for the case in which the change in pressure along the channel is zero.

See solution for Problem 6.83 to obtain

$$q = -\frac{2}{3}\frac{\mathcal{P} h^3}{\mu}$$

where q is the discharge per unit width and $\mathcal{P} = \frac{\partial p}{\partial y} + \rho g$. Thus,

$$\frac{\partial p}{\partial y} + \rho g = -\frac{3}{2}\frac{\mu q}{h^3}$$

or

$$\frac{\partial p}{\partial y} = -\frac{3}{2}\frac{\mu q}{h^3} - \rho g$$

For $\frac{\partial p}{\partial y} = 0$

$$\underline{\underline{q = -\frac{2}{3}\frac{\rho g h^3}{\mu}}}$$

(Note: The negative sign indicates that the direction of flow must be downward to create a zero pressure gradient.)