6.74

6.74 Oil (SAE 30) at 15.6 °C flows steadily between fixed, horizontal, parallel plates. The pressure drop per unit length along the channel is 20 kPa/m, and the distance between the plates is 4mm. The flow is laminar. Determine: **(a)** the volume rate of flow (per meter of width), **(b)** the magnitude and direction of the shearing stress acting on the bottom plate, and **(c)** the velocity along the centerline of the channel.

(a) $$q = \frac{2h^3}{3\mu}\frac{\Delta p}{\ell} \qquad \text{(Eq. 6.136)}$$

For $h = \frac{4\,mm}{2} = 2\times10^{-3}\,m$, $\mu = 0.38\,\frac{N\cdot s}{m^2}$, and $\frac{\Delta p}{\ell} = 20\times10^3\,\frac{N}{m^3}$,

$$q = \frac{2\,(2\times10^{-3}\,m)^3\,(20\times10^3\,\frac{N}{m^3})}{3\,(0.38\,\frac{N\cdot s}{m^2})} = \underline{\underline{2.81\times10^{-4}\,\frac{m^2}{s}}}$$

(b) $$\tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \text{(Eq. 6.125d)}$$

Since
$$u = \frac{1}{2\mu}\frac{\partial p}{\partial x}(y^2 - h^2) \qquad \text{(Eq. 6.134)}$$
and
$$v = 0$$
it follows that
$$\frac{\partial u}{\partial y} = \frac{1}{2\mu}\frac{\partial p}{\partial x}(2y) \qquad \frac{\partial v}{\partial x} = 0$$
and therefore
$$\tau_{yx} = \frac{\partial p}{\partial x}(y)$$
At the bottom plate, $y = -h$, and since $\frac{\partial p}{\partial x} = -\frac{\Delta p}{\ell}$,
$$\tau_{yx} = \frac{\Delta p}{\ell}(h) = (20\times10^3\,\frac{N}{m^3})(2\times10^{-3}\,m)$$
$= \underline{\underline{40\,\frac{N}{m^2} \text{ acting in the direction of flow}}}$

(c)
$$u_{max} = \frac{3}{2}V \qquad \text{(Eq. 6.138)}$$
$$= \frac{3}{2}\left(\frac{q}{2h}\right) = \frac{3}{2}\,\frac{(2.81\times10^{-4}\,\frac{m^2}{s})}{(2)(2\times10^{-3}\,m)} = \underline{\underline{0.105\,\frac{m}{s}}}$$