6.19

6.19 In a certain steady, two-dimensional flow field the fluid density varies linearly with respect to the coordinate x; that is, $\rho = Ax$ where A is a constant. If the x component of velocity u is given by the equation $u = y$, determine an expression for v.

For a variable density flow,

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} \qquad (\text{Eq. } 6.29)$$

With

$$\rho u = (Ax)(y) = Axy$$

it follows that

$$\frac{\partial(\rho u)}{\partial x} = Ay$$

Thus,

$$\frac{\partial(\rho v)}{\partial y} = -Ay \qquad (1)$$

Integrate Eq. (1) with respect to y to obtain

$$\int d(\rho v) = -\int Ay\,dy + f_1(x)$$

or

$$\rho v = -\frac{Ay^2}{2} + f_1(x)$$

With $\rho = Ax$

$$v = -\left(\frac{1}{Ax}\right)\left(\frac{Ay^2}{2}\right) + \frac{f_1(x)}{Ax}$$

or

$$v = -\frac{y^2}{2x} + f(x)$$

where $f(x)$ is an arbitrary function of x.