6.15

6.15 The velocity components for an incompressible, plane flow are

$$v_r = Ar^{-1} + Br^{-2}\cos\theta$$

$$v_\theta = Br^{-2}\sin\theta$$

where A and B are constants. Determine the corresponding stream function.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = -\frac{\partial\psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that for the velocity distribution given,

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta} = Ar^{-1} + Br^{-2}\cos\theta \qquad (1)$$

$$\frac{\partial\psi}{\partial r} = -Br^{-2}\sin\theta \qquad (2)$$

Integrate Eq. (1) with respect to θ to obtain

$$\int d\psi = \int (A + Br^{-1}\cos\theta)\, d\theta + f_1(r)$$

or

$$\psi = A\theta + Br^{-1}\sin\theta + f_1(r) \qquad (3)$$

Similarly, integrate Eq. (2) with respect to r to obtain

$$\int d\psi = -\int Br^{-2}\sin\theta\, dr + f_2(\theta)$$

or

$$\psi = Br^{-1}\sin\theta + f_2(\theta) \qquad (4)$$

Thus, to satisfy both Eqs. (3) and (4)

$$\psi = \underline{\underline{A\theta + Br^{-1}\sin\theta + C}}$$

where C is an arbitrary constant.