6.8

6.8 An incompressible viscous fluid is placed between two large parallel plates as shown in Fig. P6.8. The bottom plate is fixed and the upper plate moves with a constant velocity, U. For these conditions the velocity distribution between the plates is linear, and can be expressed as

$$u = U\frac{y}{b}$$

Determine: **(a)** the volumetric dilatation rate, **(b)** the rotation vector, **(c)** the vorticity, and **(d)** the rate of angular deformation.

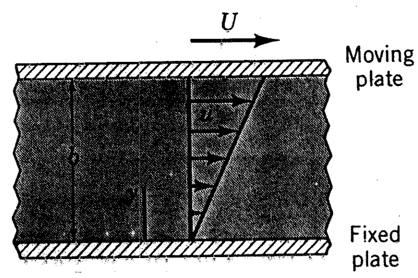

FIGURE P6.8

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \underline{\underline{0}}$

(b) For velocity distribution given,

$$\vec{\omega} = \omega_z \hat{k}$$

and

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -\frac{U}{2b}$$

Thus,

$$\vec{\omega} = \underline{\underline{-\frac{U}{2b}\hat{k}}}$$

(c) $$\vec{\zeta} = 2\vec{\omega} = \underline{\underline{-\frac{U}{b}\hat{k}}}$$

(d) $$\dot{\gamma} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \qquad \text{(Eq. 6.18)}$$

Thus,

$$\dot{\gamma} = \underline{\underline{\frac{U}{b}}}$$