

6.5

6.5 Determine an expression for the vorticity of the flow field described by

$$\mathbf{V} = -xy^3 \hat{i} + y^4 \hat{j}$$

Is the flow irrotational?

$$\vec{\mathcal{F}} = 2\vec{\omega} \quad (\text{Eq. 6.17})$$

From expression for velocity, $u = -xy^3$, $v = y^4$, and $w = 0$, and with

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \quad (\text{Eq. 6.13})$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \quad (\text{Eq. 6.14})$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad (\text{Eq. 6.12})$$

it follows that

$$\omega_x = 0, \quad \omega_y = 0, \quad \text{and} \quad \omega_z = \frac{1}{2} [0 - (-3xy^2)] = \frac{3}{2} xy^2$$

Thus,

$$\begin{aligned} \vec{\mathcal{F}} &= 2 (\omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}) \\ &= 2 \left[(0) \hat{i} + (0) \hat{j} + \left(\frac{3}{2} xy^2\right) \hat{k} \right] \\ &= \underline{\underline{3xy^2 \hat{k}}} \end{aligned}$$

Since $\vec{\mathcal{F}}$ is not zero everywhere the flow is not irrotational. No.