4.41

4.41 A fluid flows past a circular cylinder of radius a with an upstream speed of V_0 as shown in Fig. P4.41. A more advanced theory indicates that if viscous effects are negligible, the velocity of the fluid along the surface of the cylinder is given by $V = 2V_0 \sin\theta$. Determine the streamline and normal components of acceleration on the surface of the cylinder as a function of V_0, a, and θ.

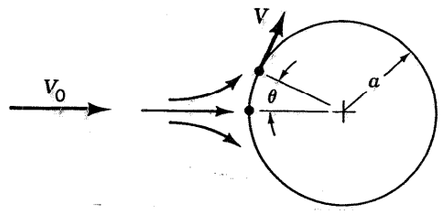

FIGURE P4.41

$$a_n = \frac{V^2}{R} = \frac{(2V_0 \sin\theta)^2}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin^2\theta}}$$

and

$$a_s = V\frac{\partial V}{\partial s} = V\frac{\partial V}{\partial \theta}\frac{\partial \theta}{\partial s}, \text{ where } \frac{\partial V}{\partial \theta} = 2V_0\cos\theta \text{ and } s = a\theta$$

or $\frac{\partial \theta}{\partial s} = \frac{1}{a}$

Thus,

$$a_s = (2V_0\sin\theta)(2V_0\cos\theta)\frac{1}{a} = \underline{\underline{\frac{4V_0^2}{a}\sin\theta\cos\theta}}$$

4.42*

4.42* Use the results of Problem 4.41 to plot graphs of a_s and a_n for $0 \le \theta \le 90°$ with $V_0 =$ 10 m/s and $a = 0.01$, 0.10, 1.0, and 10.0 m.

From Problem 4.41, $a_n = \frac{4V_0^2}{a}\sin^2\theta$ and $a_s = \frac{4V_0^2}{a}\sin\theta\cos\theta$. These results with $V_0 = 10\frac{m}{s}$ and $a = 0.01, 0.10, 1.0,$ and 10.0 m are plotted below.

	a = 0.01 m	a = 0.10 m	a = 1.0 m	a = 10 m	a = 0.10 m	a = 0.10 m	a = 1.0 m	a = 10 m
θ, deg	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_s, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2	a_n, ft/s^2
0	0	0	0	0.00	0	0	0	0.00
5	3473	347	35	3.47	304	30	3	0.30
10	6840	684	68	6.84	1206	121	12	1.21
15	10000	1000	100	10.00	2679	268	27	2.68
20	12856	1286	129	12.86	4679	468	47	4.68
25	15321	1532	153	15.32	7144	714	71	7.14
30	17321	1732	173	17.32	10000	1000	100	10.00
35	18794	1879	188	18.79	13160	1316	132	13.16
40	19696	1970	197	19.70	16527	1653	165	16.53
45	20000	2000	200	20.00	20000	2000	200	20.00
50	19696	1970	197	19.70	23473	2347	235	23.47
55	18794	1879	188	18.79	26840	2684	268	26.84
60	17321	1732	173	17.32	30000	3000	300	30.00
65	15321	1532	153	15.32	32856	3286	329	32.86
70	12856	1286	129	12.86	35321	3532	353	35.32
75	10000	1000	100	10.00	37321	3732	373	37.32
80	6840	684	68	6.84	38794	3879	388	38.79
85	3473	347	35	3.47	39696	3970	397	39.70
90	0	0	0	0.00	40000	4000	400	40.00

(con't)