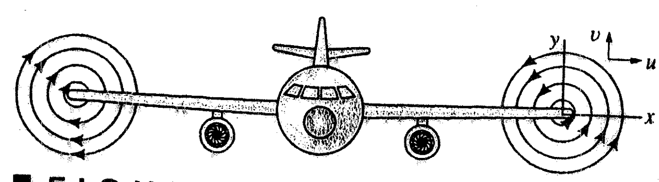
4.37

4.37 As shown in **Video V4.2** and **Fig. P4.37**, a flying airplane produces swirling flow near the end of its wings. In certain circumstances this flow can be approximated by the velocity field $u = -Ky/(x^2 + y^2)$ and $v = Kx/(x^2 + y^2)$, where K is a constant depending on various parameter associated with the airplane (i.e., its weight, speed) and x and y are measured from the center of the swirl. **(a)** Show that for this flow the velocity is inversely proportional to the distance from the origin. That is, $V = K/(x^2 + y^2)^{1/2}$. **(b)** Show that the streamlines are circles.



■ **FIGURE P4.37**

(a) $$V = \sqrt{u^2 + v^2} = \left[\frac{(-Ky)^2}{(x^2+y^2)^2} + \frac{(Kx)^2}{(x^2+y^2)^2}\right]^{1/2} = \frac{K}{\sqrt{x^2+y^2}}$$

or

$$V = \frac{K}{r}, \quad \text{where } r = \sqrt{x^2+y^2}$$

(b) Streamlines are given by $$\frac{dy}{dx} = \frac{v}{u} = \frac{\dfrac{Kx}{(x^2+y^2)}}{\dfrac{-Ky}{(x^2+y^2)}} = -\frac{x}{y}$$

Thus,

$y\,dy = -x\,dx$ which when integrated gives

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C_1$, where C_1 is a constant.

or

$$x^2 + y^2 = \text{Constant}$$