4.27

4.27 A nozzle is designed to accelerate the fluid from V_1 to V_2 in a linear fashion. That is, $V = ax + b$, where a and b are constants. If the flow is constant with $V_1 = 10$ m/s at $x_1 = 0$ and $V_2 = 25$ m/s at $x_2 = 1$ m, determine the local acceleration, the convective acceleration, and the acceleration of the fluid at points (1) and (2).

With $u = ax + b$, $v = 0$, and $w = 0$ the acceleration $\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ can be written as

$$\vec{a} = a_x \hat{i} \quad \text{where} \quad a_x = u\frac{\partial u}{\partial x}. \qquad (1)$$

Since $u = V_1 = 10\frac{m}{s}$ at $x = 0$ and $u = V_2 = 25\frac{m}{s}$ at $x = 1$ we obtain

$$10 = 0 + b$$

$$25 = a + b \quad \text{so that } a = 15 \text{ and } b = 10$$

That is, $u = (15x + 10)\frac{m}{s}$, where $x \sim m$, so that from Eq.(1)

$$a_x = (15x + 10)\frac{m}{s}\left(15\frac{1}{s}\right) = \underline{\underline{(225x + 150)\frac{m}{s^2}}}$$

Note: The local acceleration is zero, $\underline{\underline{\frac{\partial \vec{V}}{\partial t} = 0}}$, and the convective acceleration is $u\frac{\partial u}{\partial x}\hat{i} = \underline{\underline{(225x + 150)\hat{i}\,\frac{m}{s^2}}}$

At $x = 0$, $\vec{a} = \underline{\underline{150\hat{i}\,\frac{m}{s^2}}}$; at $x = 1\,m$, $\vec{a} = \underline{\underline{375\hat{i}\,\frac{m}{s^2}}}$