3.31

3.31 Water flows through the pipe contraction shown in Fig. P3.31. For the given 0.2-m difference in the manometer level, determine the flowrate as a function of the diameter of the small pipe, D.

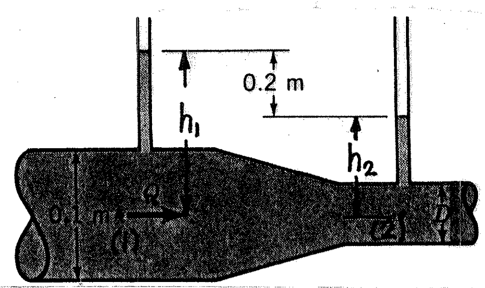

FIGURE P3.31

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad \text{with } A_1V_1 = A_2V_2$$

$$\text{or} \quad V_2 = \frac{\left(\frac{\pi}{4}D_1^2\right)}{\left(\frac{\pi}{4}D_2^2\right)}V_1 = \left(\frac{0.1}{D}\right)^2 V_1$$

Thus, with $z_1 = z_2$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g}$$

but

$p_1 = \gamma h_1$ and $p_2 = \gamma h_2$ so that $p_1 - p_2 = \gamma(h_1 - h_2) = 0.2\gamma$

Thus,

$$\frac{0.2\gamma}{\gamma} = \frac{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]V_1^2}{2g} \quad \text{or} \quad V_1 = \sqrt{\frac{0.2\,(2g)}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

and

$$Q = A_1V_1 = \frac{\pi}{4}(0.1)^2\sqrt{\frac{0.2\,(2\,(9.81))}{\left[\left(\frac{0.1}{D}\right)^4 - 1\right]}}$$

or

$$\underline{\underline{Q = \frac{0.0156\,D^2}{\sqrt{(0.1)^4 - D^4}}}} \; \frac{m^3}{s} \quad \text{when } D \sim m$$