3.11

3.11 It can be shown that if viscous and gravitational effects are neglected, the fluid velocity along the surface of a circular cylinder of radius a is $V = 2V_0 \sin\theta$, where V_0 is the upstream velocity and $s = a\theta$ is the distance measured along the streamline that coincides with the cylinder (see Fig. P3.11). For a fluid of density ρ, determine the pressure gradient in the radial direction, $\partial p/\partial r$, on the surface of the cylinder. Assume the axis of the cylinder is vertical. Is $\partial p/\partial r$ positive or negative? Explain physically. For what θ is $\partial p/\partial r$ the maximum? Explain why.

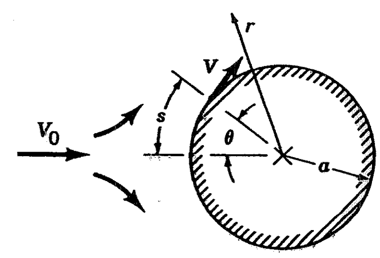


FIGURE P3.11

$$-\gamma\frac{dz}{dn} - \frac{\partial p}{\partial n} = \frac{\rho V^2}{\mathcal{R}} \quad \text{with } \frac{\partial}{\partial n} = \frac{\partial}{\partial r}\frac{\partial r}{\partial n} = -\frac{\partial}{\partial r} \text{ and } \frac{dz}{dn} = 0$$

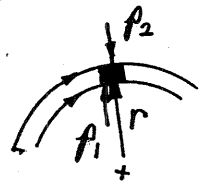
so that

$$\frac{\partial p}{\partial r} = \frac{\rho V^2}{\mathcal{R}} = \frac{\rho V^2}{a} \quad \text{where } V = 2V_0 \sin\theta \text{ and } \mathcal{R} = a$$

Thus,

$$\underline{\underline{\frac{\partial p}{\partial r} = 4\rho V_0^2 \sin^2\theta / a}}$$

Note that for any location (i.e. θ) it follows that $\underline{\underline{\frac{\partial p}{\partial r} > 0}}$, except at $\theta = 0$ or $\theta = 180$ deg where $\frac{\partial p}{\partial r} = 0$



Must have $p_2 > p_1$ if fluid is to follow a curved path, except where $V = 0$ (as at $\theta = 0$ or $\theta = 180$ deg.)

Maximum $\frac{\partial p}{\partial r}$ occurs at $\theta = 90$ deg (i.e. maximum of $\sin^2\theta$) since that is the location of maximum normal acceleration.