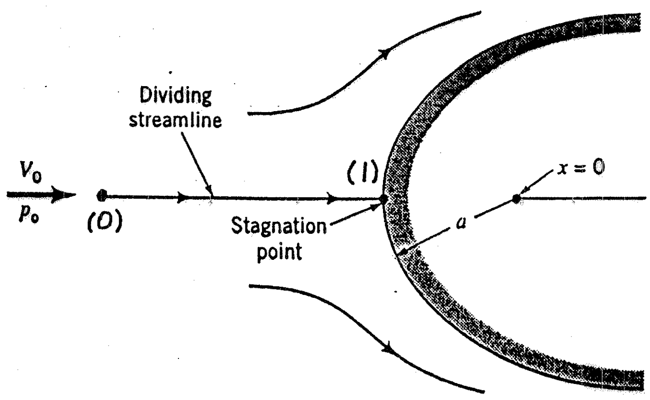
3.3

3.3 An incompressible fluid with density ρ flows steadily past the object shown in Video V3.3 and Fig. P3.3. The fluid velocity along the horizontal dividing streamline $(-\infty \le x \le -a)$ is found to be $V = V_0(1 + a/x)$, where a is the radius of curvature of the front of the object and V_0 is the upstream velocity. (a) Determine the pressure gradient along this streamline. (b) If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \le x \le -a$. (c) Show from the result of part (b) that the pressure at the stagnation point $(x = -a)$ is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.



■ FIGURE P3.3

(a) $\frac{dp}{ds} = -\rho V \frac{dV}{ds}$ where $V = V_0\left(1 + \frac{a}{x}\right)$

Thus, $\frac{dV}{ds} = \frac{dV}{dx} = -\frac{V_0 a}{x^2}$

or

$$\frac{dp}{ds} = \frac{dp}{dx} = -\rho V_0\left(1+\frac{a}{x}\right)\left(-\frac{V_0 a}{x^2}\right) = \underline{\underline{\rho a V_0^2\left(\frac{1}{x^2} + \frac{a}{x^3}\right)}}$$

(b) $$\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}\,dx = \rho a V_0^2 \int_{-\infty}^{x}\left(\frac{1}{x^2} + \frac{a}{x^3}\right)dx$$ Note: $p = p_0$ at $x = -\infty$

or

$$p - p_0 = \rho a V_0^2\left[-\frac{1}{x} - \frac{1}{2}\frac{a}{x^2}\right]_{-\infty}^{x}$$

Thus,

$$\underline{\underline{p = p_0 - \rho a V_0^2\left[\frac{1}{x} + \frac{a}{2x^2}\right]}}$$

(c) From part (b), when $x = -a$

$$p\Big|_{x=-a} = p_0 - \rho a V_0^2\left[-\frac{1}{a} + \frac{a}{2a^2}\right] = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$

From the Bernoulli equation $p_0 + \frac{1}{2}\rho V_0^2 = p_1 + \frac{1}{2}\rho V_1^2$

where

$$V_1 = V\Big|_{x=-a} = V_0\left(1 + \frac{a}{(-a)}\right) = 0$$

Thus, $p_1 = p_0 + \frac{1}{2}\rho V_0^2$ as expected.