3.1

3.1 Water flows steadily through the variable area horizontal pipe shown in Fig. P3.1. The velocity is given by $\mathbf{V} = 10(1 + x)\hat{\mathbf{i}}$ ft/s, where x is in feet. Viscous effects are neglected. **(a)** Determine the pressure gradient, $\partial p/\partial x$, (as a function of x) needed to produce this flow. **(b)** If the pressure at section (1) is 50 psi, determine the pressure at (2) by: (i) integration of the pressure gradient obtained in **(a)**; (ii) application of the Bernoulli equation.

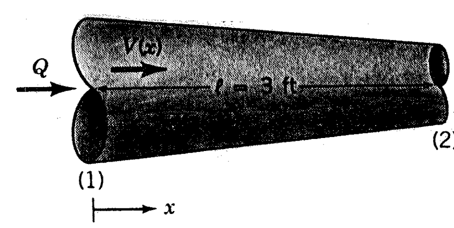

FIGURE P3.1

(a) $-\gamma \sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $V = 10(1+x)$ ft/s

$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial s}$ or $\frac{\partial p}{\partial x} = -\rho V \frac{\partial V}{\partial x} = -\rho\,(10(1+x))(10)$

Thus, $\frac{\partial p}{\partial x} = -1.94 \frac{\text{slugs}}{\text{ft}^3}(10 \frac{\text{ft}}{\text{s}})^2 (1+x)$, with x in feet

$= \underline{\underline{-194(1+x) \frac{\text{lb}}{\text{ft}^3}}}$

(b)(i) $\frac{dp}{dx} = -194(1+x)$ so that $\int_{p_1 = 50\,\text{psi}}^{p_2} dp = -194 \int_{x_1=0}^{x_2=3} (1+x)\,dx$

or $p_2 = 50\,\text{psi} - 194(3 + \frac{3^2}{2}) \frac{\text{lb}}{\text{ft}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in.}^2}\right) = 50 - 10.1 = \underline{\underline{39.9\,\text{psi}}}$

(ii) $p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$ or with $z_1 = z_2$

$p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)$ where $V_1 = 10(1+0) = 10 \frac{\text{ft}}{\text{s}}$

$V_2 = 10(1+3) = 40 \frac{\text{ft}}{\text{s}}$

Thus,

$p_2 = 50\,\text{psi} + \frac{1}{2}(1.94 \frac{\text{slugs}}{\text{ft}^3})(10^2 - 40^2)\frac{\text{ft}^2}{\text{s}^2}\left(\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right) = \underline{\underline{39.9\,\text{psi}}}$