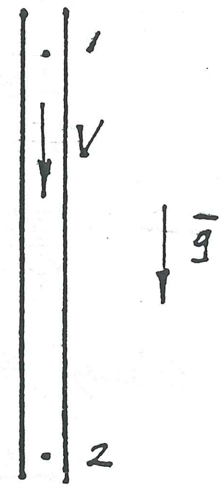
PROBLEM 8.25

GIVEN 20°C water flows down a vertical pipe where there is no pressure drop.



FIND Range of pipe diameters D (if any) for which the flow is definitely laminar.

SOLUTION Apply the mechanical energy equation from point 1 to point 2.

$$\frac{p_1}{\rho} + \frac{\alpha_1 V_1^2}{2} + g z_1 - w_s = \frac{p_2}{\rho} + \frac{\alpha_2 V_2^2}{2} + g z_2 + g h_L .$$

The problem statement gives $p_1 = p_2$ and $w_s = 0$. Assuming constant water density, the continuity equation gives $V_1 = V_2$. Let $z_1 - z_2 = h$. Assume fully developed flow so $\alpha_1 = \alpha_2 = 2.0$. This gives

$$gh = g h_L = f \frac{L}{D} \frac{V^2}{2} = f \frac{L}{D} \frac{8Q^2}{\pi^2 D^4} = \frac{8 f L Q^2}{\pi^2 D^5} . \quad (1)$$

The maximum Reynolds number for definitely laminar flow is about ~~2300~~ 2100 so

$$\frac{VD}{\nu} = \frac{4Q}{\pi D \nu} < 2100 \quad \text{or} \quad Q < 525\, \pi D \nu \quad (2)$$

and

$$f = \frac{64}{Re} = \frac{16 \pi D \nu}{Q} . \quad (3)$$

Equations (1) and (3) give

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$$gh = \frac{8fl\,Q^2}{\pi^2 D^5} = \frac{8lQ^2}{\pi^2 D^5}\left(\frac{16\pi D\nu}{Q}\right) = \frac{128 l\nu Q}{\pi D^4}$$

or

$$D^4 = \frac{128\, l\nu Q}{\pi g h}.$$

Since the pipe is vertical, $h = l$ and

$$D^4 = \frac{128\,\nu Q}{\pi g}. \qquad (4)$$

Substituting for Q from eq. (2) gives

$$D^4 \leq \frac{128\,\nu}{\pi g}\left(\cancel{575}\; 525\, \pi D \nu\right)$$

$$D \leq \sqrt[3]{\cancel{73{,}600}\; 67200\; \frac{\nu^2}{g}}.$$

~~Using Table A.3~~ The numerical values give

$$D \leq \sqrt[3]{\frac{\cancel{73{,}600}\; 67200\;(1.0 \times 10^{-6}\ \mathrm{m^2/s})^2}{(9.807\ \frac{\mathrm{m}}{\mathrm{s}^2})}}$$

$$\boxed{D \leq \cancel{0.00196\ \mathrm{m} = 1.96\ \mathrm{mm}}}$$

$$0.00190\ \mathrm{m} = 1.90\ \mathrm{mm}$$

(JIR)