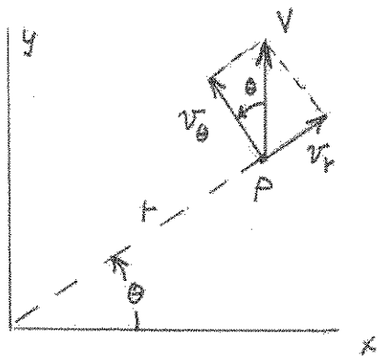
6.17

6.17 For a certain two-dimensional flow field

$$u = 0$$

$$v = V$$

(a) What are the corresponding radial and tangential velocity components? **(b)** Determine the corresponding stream function expressed in Cartesian coordinates and in cylindrical polar coordinates.



(a) At an arbitrary point P (see figure)

$$v_r = V \sin\theta$$

$$v_\theta = V \cos\theta$$

(b) Since

$$u = \frac{\partial \psi}{\partial y} = 0 \qquad v = -\frac{\partial \psi}{\partial x} = V$$

it follows that ψ is not a function of y and

$$\psi = -Vx + C$$

where C is an arbitrary constant.

Also, with $x = r\cos\theta$

$$\psi = -Vr\cos\theta + C$$

Check this result:

$$v_\theta = -\frac{\partial \psi}{\partial r} = -(-V\cos\theta) = V\cos\theta$$

and

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = \frac{1}{r}(Vr\sin\theta) = V\sin\theta$$, which checks with part (a).