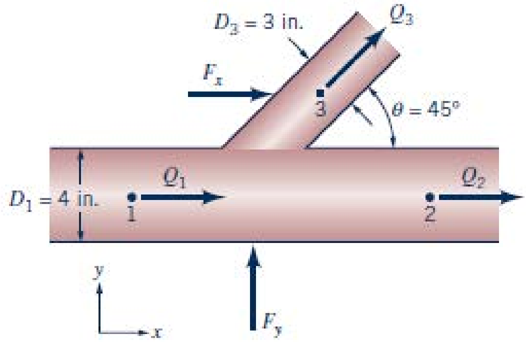
Problem 5.46

Figure P5.46 shows a lateral pipe fitting. This particular fitting has a mainline diameter of 4.0 in. The diameter of the lateral is 3.0 in., and the lateral angle is 45°; 60 °F water is flowing in the lateral. Measurements show that the pressure at point 1 is 34.0 psig, the pressure at point 2 is 35.0 psig, the pressure at point 3 is 33.5 psig, and the flow rate at point 2 is 1.0 ft^3/s. Determine the horizontal and vertical force components (*Fx* and *Fy*) required to hold the lateral fitting stationary. Neglect gravity. $Q_1 = 1.63\ ft^3/s$.



Solution

First apply the continuity equation

$$Q_1 = Q_2 + Q_3 \Rightarrow Q_3 = Q_1 - Q_2 = 1.63\frac{ft^3}{s} - 1.0\frac{ft^3}{s} = 0.63\frac{ft^3}{s}$$

For the "*x*" direction

$$\sum F_x = F_x + p_1A_1 - p_2A_2 - p_3A_3\cos 45^o = \rho Q_2V_2 + \rho Q_3V_3\cos 45^o - \rho Q_1V_1$$

so

$$F_x = \rho Q_2V_2 + \rho Q_3V_3\cos 45^o - \rho Q_1V_1 + p_2A_2 + p_3A_3\cos 45^o - p_1A_1$$

but $A = \frac{\pi}{4}D^2$ and $V = \frac{Q}{A}$ so $V = \frac{4Q}{\pi D^2}$

substituting $F_x = \frac{4\rho Q_2^2}{\pi D_2^2} + \frac{4\rho Q_3^2}{\pi D_3^2}\cos 45^o - \frac{4\rho Q_1^2}{\pi D_2^2} + p_2\frac{\pi}{4}D_2^2 + p_3\frac{\pi}{4}D_3^2\cos 45^o - p_1\frac{\pi}{4}D_1^2$

gathering terms and substituting numerical values and conversion factors gives

$$F_x = \frac{4\rho}{\pi}\left(\frac{Q_2^2}{D_2^2} + \frac{Q_3^2}{D_3^2}\cos 45^o - \frac{Q_1^2}{D_2^2}\right) + \frac{\pi}{4}\left(p_2D_2^2 + p_3D_3^2\cos 45^o - p_1D_1^2\right)$$

$$F_x = \frac{4\times 1.94\frac{slugs}{ft^3}}{\pi}\left(\frac{1.0^2\frac{ft^6}{s^2}}{(0.333\,ft)^2} + \frac{0.63^2\frac{ft^6}{s^2}}{(0.25\,ft)^2}\cos 45^o - \frac{1.63^2\frac{ft^6}{s^2}}{(0.333\,ft)^2}\right) + \frac{\pi}{4}\left(35.0\frac{lb}{in^2}\times 16in^2 + 33.5\frac{lb}{in^2}\times 9in^2\cos 45^o - 34.0\frac{lb}{in^2}\times 16in^2\right)$$

$$\underline{\underline{F_x = +154.2lb}}\ \text{(acts to the right)}$$

For the "*y*" direction

$$\sum F_y = F_y - p_3A_3 = \rho Q_3V_3\sin 45^o - 0 \Rightarrow F_y = p_3A_3 + \rho Q_3V_3\sin 45^o$$

The weight of the fitting and the water in it have been neglected. Substituting as before

$$F_y = p_3 \frac{\pi}{4} D_3^2 + \frac{4\rho}{\pi} \frac{Q_3^2}{D_3^2} \sin 45^o$$

$$F_y = \frac{4 \times 1.94 \dfrac{slugs}{ft^3}}{\pi} \left(\frac{0.63^2 \dfrac{ft^6}{s^2}}{(0.25\, ft)^2} \sin 45^o \right) + \frac{\pi}{4} \left(33.5 \frac{lb}{in^2} \times 9 in^2 \sin 45^o \right)$$

$$\underline{\underline{F_y = +28.0 lb}} \text{ (acts up)}$$