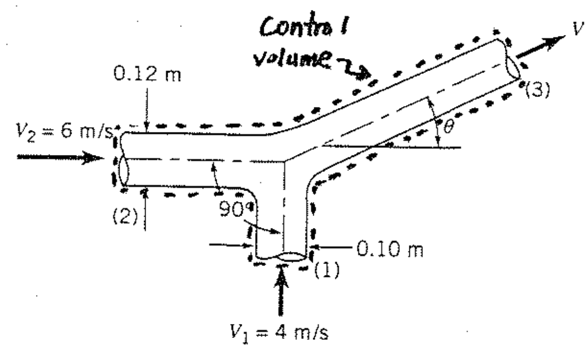
5.128

5.128 Two water jets collide and form one homogeneous jet as shown in Fig. P5.134. (a) Determine the speed, V, and direction, θ, of the combined jet. (b) Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.134

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4 \frac{m}{s}\right)^2 \frac{\pi (0.1m)^2}{4}}{\left(6 \frac{m}{s}\right)^2 \frac{\pi (0.12m)^2}{4}} = 0.3086$$

so

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \left(V_1 A_1 + V_2 A_2\right)} = \frac{V_2^2 d_2^2}{\cos\theta \left(V_1 d_1^2 + V_2 d_2^2\right)}$$

Thus

$$V_3 = \frac{\left(6 \frac{m}{s}\right)^2 (0.12m)^2}{(\cos 17.2^\circ)\left[\left(4 \frac{m}{s}\right)(0.1m)^2 + \left(6 \frac{m}{s}\right)(0.12m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29 \frac{m}{s}}}$$

(con't)

5.128 (con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1 + \frac{V_1^2}{2}\right)\dot{m}_1 - \left(\check{u}_2 + \frac{V_2^2}{2}\right)\dot{m}_2 + \left(\check{u}_3 + \frac{V_3^2}{2}\right)\dot{m}_3 = 0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1 - \dot{m}_2 + \dot{m}_3 = 0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3 - \check{u}_1) + \dot{m}_2(\check{u}_3 - \check{u}_2) = \dot{m}_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \dot{m}_2\left(\frac{V_2^2 - V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss} = \rho V_1 A_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \rho V_2 A_2\left(\frac{V_2^2 - V_3^2}{2}\right)$$

or

$$\text{rate of loss} = \frac{\rho\pi}{4}\left[d_1^2 V_1\left(\frac{V_1^2 - V_3^2}{2}\right) + d_2^2 V_2\left(\frac{V_2^2 - V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss} = \frac{\left(999\frac{kg}{m^3}\right)(3.14)\left(1\frac{N\cdot s^2}{kg\cdot m}\right)}{4}\left\{(0.10\,m)^2\left(4\frac{m}{s}\right)\left[\frac{\left(4\frac{m}{s}\right)^2 - \left(4.29\frac{m}{s}\right)^2}{2}\right] + (0.12\,m)^2\left(6\frac{m}{s}\right)\left[\frac{\left(6\frac{m}{s}\right)^2 - \left(4.29\frac{m}{s}\right)^2}{2}\right]\right\}$$

and

$$\text{rate of loss} = \underline{\underline{558\ \frac{N\cdot m}{s}}}$$