5.114

5.114 The turbine shown in Fig. P5.121 develops 100 hp when the flowrate of water is 20 ft^3/s. If all losses are negligible, determine **(a)** the elevation h, **(b)** the pressure difference across the turbine, and **(c)** the flowrate expected if the turbine were removed.

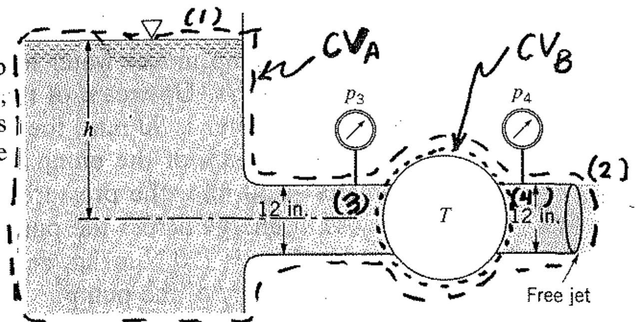

FIGURE P5.121

(a) Using control volume A and the energy equation (Eq. 5.84) we get:

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\gamma}}^{\,0} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 + h_s - \cancel{h_\ell}^{\,0} \qquad (1)$$

For a turbine, $h_T = -h_s$ and from Eq. 5.85 we get:

$$h_T = \frac{\dot{W}_{\substack{shaft \\ net\,out}}}{\gamma Q} = \frac{(100\ hp)\left(550\ \frac{ft \cdot lb}{s \cdot hp}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)\left(20\ \frac{ft^3}{s}\right)} = 44.1\ ft$$

Since $Q = AV$ we have

$$V_2 = \frac{Q}{A_2} = \frac{20\ \frac{ft^3}{s}}{\frac{\pi d_2^2}{4}} = \frac{20\ \frac{ft^3}{s}}{\frac{\pi}{4}\left(\frac{12\ in.}{12\ \frac{in.}{ft}}\right)^2} = 25.5\ \frac{ft}{s}$$

Then from eq. 1

$$z_1 - z_2 = h = \frac{V_2^2}{2g} - h_s = \frac{\left(25.5\ \frac{ft}{s}\right)^2}{2\left(32.2\ \frac{ft}{s^2}\right)} + 44.1\ ft = \underline{\underline{54.1}}\ ft$$

(b) For control volume B the energy equation yields

$$p_3 - p_4 = \gamma h_T = \left(62.4\ \frac{lb}{ft^3}\right)(44.1\ ft) = \underline{\underline{2.75}}\ \frac{lb}{ft^2}$$

(c) Since $Q = VA = V_2 A_2$, if we knew value of V_2 with the turbine removed, we could calculate Q with the turbine removed. Without the turbine, Eq. (1) reduces to

$$\frac{V_2^2}{2g} = z_1 - z_2 = h$$

and

$$V_2 = \sqrt{2gh} = \sqrt{2\left(32.2\ \frac{ft}{s^2}\right)(54.1\ ft)} = 59\ \frac{ft}{s}$$

Thus

$$Q_{\substack{w/o \\ turbine}} = \frac{\pi d_2^2}{4} V_2 = \frac{\pi}{4}\left(\frac{12\ in.}{12\ \frac{in.}{ft}}\right)^2\left(59\ \frac{ft}{s}\right) = \underline{\underline{46.3}}\ \frac{ft^3}{s}$$