PROBLEM 4.23

PROBLEM ~~5.3~~

GIVEN Velocity components

$$u = ax + \frac{bx}{x^2y^2} \quad \text{and} \quad v = ay + \frac{by}{x^2y^2}$$

where a and b are constants.

FIND Accelerations a_x and a_y.

SOLUTION Since the flow is spatially two-dimensional (x, y) and steady, Equations (3.52) and (3.53) give

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \quad \text{and} \quad a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}.$$

Substituting into the equation for a_x gives

$$a_x = \left(ax + \frac{b}{xy^2}\right)\frac{\partial}{\partial x}\left(ax + \frac{b}{xy^2}\right) + \left(ay + \frac{b}{x^2y}\right)\frac{\partial}{\partial y}\left(ax + \frac{b}{xy^2}\right)$$

$$= \left(ax + \frac{b}{xy^2}\right)\left(a - \frac{b}{x^2y^2}\right) + \left(ay + \frac{b}{x^2y}\right)\left(-\frac{2b}{xy^3}\right)$$

$$= a^2x + \frac{ab}{xy^2} - \frac{ab}{xy^2} - \frac{b^2}{x^3y^4} - \frac{2ab}{xy^2} - \frac{2b^2}{x^3y^4}$$

$$= a^2x - \frac{2ab}{xy^2} - \frac{3b^2}{x^3y^4} = x\left(a^2 - \frac{2ab}{x^2y^2} - \frac{3b^2}{x^4y^4}\right)$$

or

$$\boxed{a_x = x\left(a - \frac{3b}{x^2y^2}\right)\left(a + \frac{b}{x^2y^2}\right).}$$

Substituting into the equation for a_y gives

$$a_y = \left(ax + \frac{b}{xy^2}\right)\frac{\partial}{\partial x}\left(ay + \frac{b}{x^2y}\right) + \left(ay + \frac{b}{x^2y}\right)\frac{\partial}{\partial y}\left(ay + \frac{b}{x^2y}\right)$$

$$= \left(ax + \frac{b}{xy^2}\right)\left(\frac{-2b}{x^3y}\right) + \left(ay + \frac{b}{x^2y}\right)\left(a - \frac{b}{x^2y^2}\right)$$

PROBLEM 4.23

4.23

PROBLEM ~~5.3~~

$$= \frac{2ab}{x^2y} - \frac{2b^2}{x^4y^3} + a^2y + \frac{ab}{x^2y} - \frac{ab}{x^2y} - \frac{b^2}{x^4y^3}$$

$$= a^2y - \frac{2ab}{x^2y} - \frac{3b^2}{x^4y^3} = y\left(a^2 - \frac{2ab}{x^2y^2} - \frac{3b^2}{x^4y^4}\right)$$

or

$$\boxed{a_y = y\left(a - \frac{3b}{x^2y^2}\right)\left(a + \frac{b}{x^2y^2}\right).}$$

~~(MQ)~~