7.9 The excess pressure inside a bubble (discussed in Chapter 1) is known to be dependent on bubble radius and surface tension. After finding the pi terms, determine the variation in excess pressure if we (a) double the radius and (b) double the surface tension.

Given $\Delta p = f(R, \sigma)$, where $\Delta p \doteq \frac{F}{L^2} = \frac{M}{LT^2}$, $R \doteq L$, and $\sigma \doteq \frac{F}{L} = \frac{M}{T^2}$

Consider the (MLT) units so that
$k - r = 3 - 3 = 0$ since there 3 variables and 3 dimensions.
According to this, there should be $k - r = 0$ pi terms !?
However, if we consider the (FLT) units we see that it takes only F and L, T is not needed, so that $r = 2$.
Hence, $k - r = 3 - 2 = 1$, so only 1 pi term is needed.

That is, $\Pi_1 = $ constant
To determine Π_1, consider
$\Pi_1 = \Delta p \, R^a \sigma^b$ or
$\Delta p \, R^a \sigma^b \doteq \frac{F}{L^2} L^a \left(\frac{F}{L}\right)^b = F^{1+b} L^{-2+a-b}$
Thus:
$F: \quad 1 + b = 0$
$L: \quad a - b - 2 = 0$
or $b = -1$ and $a = b + 2 = 1$
Hence $\Pi_1 = \frac{\Delta p \, R}{\sigma}$ or $\frac{\Delta p \, R}{\sigma} = C$, where $C = $ constant.

or
$$\Delta p = \frac{C \sigma}{R} \qquad\qquad (1)$$

(a) If R is doubled, Δp is reduced by half. (see Eq. (1))

(b) If σ is doubled, Δp is doubled. (see Eq. (1))