5.118

5.118 Water is pumped from the tank shown in Fig. P5.118*a*. The head loss is known to be $1.2\ V^2/2g$, where V is the average velocity in the pipe. According to the pump manufacturer, the relationship between the pump head and the flowrate is as shown in Fig. P5.118 *b*: $h_p = 20 - 2000\ Q^2$, where h_p is in meters and Q is in m^3/s. Determine the flowrate, Q.

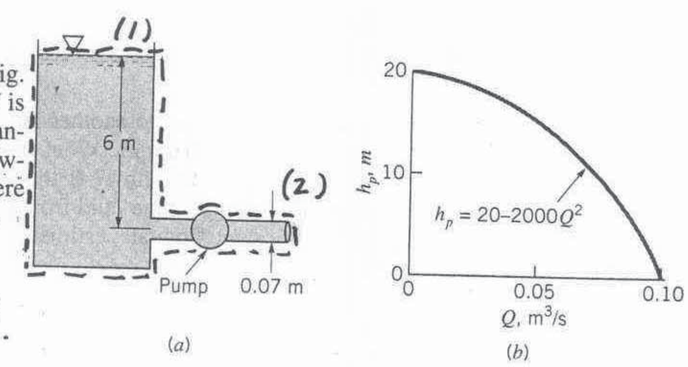

FIGURE P5.118

We want to know the flowrate Q.

For the control volume shown, application of the energy equation (Eq. 5.84) yields:

$$\cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 + h_s - h_\ell \qquad (1)$$

However

$$h_\ell = 1.2\frac{V_2^2}{2g} \qquad (2)$$

and

$$h_s = h_p = 20 - 2000\ Q^2 \qquad (3)$$

Since $Q = V_2 A_2$ we have from eq. 2

$$h_\ell = \frac{1.2}{2g}\left(\frac{Q}{A}\right)^2 \qquad (4)$$

and combining Eqs. (1), (3) and (4) we get:

$$\frac{1}{2g}\left(\frac{Q}{A_2}\right)^2 + z_2 = z_1 + 20 - 2000\ Q^2 - \frac{1.2}{2g}\left(\frac{Q}{A_2}\right)^2 \qquad (5)$$

or

$$Q^2\left(\frac{1}{2gA_2^2} + \frac{1.2}{2gA_2^2} + 2000\right) = z_1 - z_2 + 20$$

So

$$Q = \left[\frac{z_1 - z_2 + 20}{\frac{1}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + \frac{1.2}{2g\left(\frac{\pi d_2^2}{4}\right)^2} + 2000}\right]^{\frac{1}{2}} = \left[\frac{6\,m + 20\,m}{\frac{1}{(2)(9.81\frac{m}{s^2})\left[\frac{\pi(0.07m)^2}{4}\right]^2} + \frac{1.2}{2(9.81\frac{m}{s^2})\left[\frac{\pi(0.07m)^2}{4}\right]^2} + 2000}\right]^{\frac{1}{2}}$$

$$Q = \underline{\underline{0.052\ \frac{m^3}{s}}}$$