6–25 A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle $\theta = 45°$ from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is 150 cm^2 at the inlet and 25 cm^2 at the exit. The elevation difference between the centers of the exit and the inlet is 40 cm. The mass of the elbow and the water in it is 50 kg. Determine the anchoring force needed to hold the elbow in place.

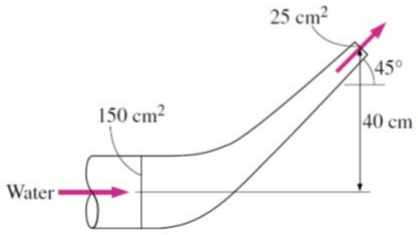

FIGURE P6–25

Mass conservation:

$$\dot{m}_1 = \dot{m}_2 = \dot{m} = 30\,\text{kg/s}$$

$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{30}{(1{,}000)(0.015)} = 2\,\text{m/s}$$

$$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{30}{(1{,}000)(0.0025)} = 12\,\text{m/s}$$

Bernoulli equation:

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

$$p_1 + \frac{1}{2}(1{,}000)(2)^2 + (1{,}000 \times 9.81)z_1 = (0) + \frac{1}{2}(1{,}000)(12)^2 + (1{,}000 \times 9.81)(z_1 + 0.4)$$

$$\therefore p_1 = \frac{1}{2}(1{,}000)(12^2 - 2^2) + (1{,}000 \times 9.81)0.4 = 73{,}924\,\text{N/m}^2 = 73.9\,\text{kPa}$$

x-momentum equation:

$$\dot{m}_2 u_2 - \dot{m}_1 u_1 = p_1 A_1 + F_x$$

or

$$\dot{m}_2(V_2 \cos 45°) - \dot{m}_1 V_1 = p_1 A_1 + F_x$$

$$\therefore F_x = \dot{m}(V_2 \cos 45° - V_1) - p_1 A_1$$

Thus,

$$F_x = (30)(12 \cos 45° - 2) - (73{,}924)(0.0150) = -914.3\,\text{N}$$

y-momentum equation:

$$\dot{m}_2 v_2 - \dot{m}_1 v_1 = -W + F_y$$

or

$$\dot{m}_2(V_2 \sin 45°) - \dot{m}_1(0) = -mg + F_y$$

$$\therefore F_y = \dot{m} V_2 \sin 45° + mg$$

Thus,

$$F_y = (30)(12) \sin 45° + (50)(9.81) = 780.7\,\text{N}$$