4.21

4.21 Determine the acceleration field for a three-dimensional flow with velocity components $u = -x$, $v = 4x^2y^2$, and $w = x - y$.

$u = -x$, $v = 4x^2y^2$, and $w = x-y$ so that

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + (-x)(-1) + 4x^2y^2(0) + (x-y)(0) = x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$= 0 + (-x)(8xy^2) + (4x^2y^2)(8x^2y) + (x-y)(0)$$

$$= -8x^2y^2 + 32x^4y^3 = 8x^2y^2(4x^2y-1)$$

and

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$= 0 + (-x)(1) + (4x^2y^2)(-1) + (x-y)(0)$$

$$= -x - 4x^2y^2$$

Thus,

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$= \underline{\underline{x\hat{\imath} + 8x^2y^2(4x^2y-1)\hat{\jmath} - (x+4x^2y^2)\hat{k}}}$$