2.10

2.10 For the great depths that may be encountered in the ocean the compressibility of seawater may become an important consideration. **(a)** Assume that the bulk modulus for seawater is constant and derive a relationship between pressure and depth which takes into account the change in fluid density with depth. **(b)** Make use of part (a) to determine the pressure at a depth of 6 km assuming seawater has a bulk modulus of 2.3×10^9 Pa, and a density of 1030 kg/m³ at the surface. Compare this result with that obtained by assuming a constant density of 1030 kg/m³.

(a)

$$\frac{dp}{dz} = -\gamma = -\rho g \qquad (\text{Eq. } 2.4)$$

Thus,

$$\frac{dp}{\rho} = -g\, dz \qquad (1)$$

If ρ is a function of p, we must determine $\rho = f(p)$ before integrating Eq.(1). Since,

$$E_v = \frac{dp}{d\rho/\rho} \qquad (\text{Eq. } 1.13)$$

then

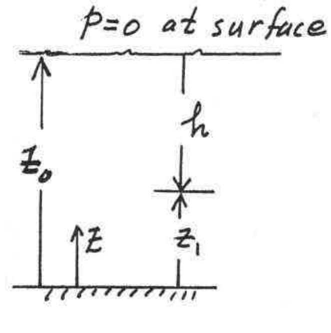
$$\int_0^p dp = E_v \int_{\rho_0}^{\rho} \frac{d\rho}{\rho}$$

so that

$$p = E_v \ln \frac{\rho}{\rho_0}$$

Thus,

$$\rho = \rho_0 e^{\frac{p}{E_v}} \qquad \text{where } \rho = \rho_0 \text{ at } p = 0$$



From Eq.(1)

$$\int_{p_1}^{0} \frac{dp}{\rho_0 e^{\frac{p}{E_v}}} = -g \int_{z_1}^{z_0} dz$$

or

$$\int_{p_1}^{0} e^{-\frac{p}{E_v}}\, dp = -\rho_0 g \int_{z_1}^{z_0} dz$$

so that

$$\underline{\underline{p = -E_v \ln\left(1 - \frac{\rho_0 g h}{E_v}\right)}} \qquad \text{where } h = z_0 - z_1 \text{, the depth below surface}$$

(con't)