**Appendix C. Calculation of Airfoil Drag Force Using the Momentum Integral Method**

Consider an experiment in which there is drag force on an airfoil that is immersed in a steady incompressible flow. The drag force can be determined from the measurement of velocity distributions far upstream and downstream of the airfoil body (figure below).

1. Velocity far upstream is the uniform flow .
2. Velocity in the wake of the body is measured with hotwire/Pitot probe to be , which is less than due to the drag of the airfoil.
3. Objective: Find the drag force per unit length (span wise) of the airfoil.

Method 1: Choose a control volume following the flow streamline.



Fig. 1 Control volume following the flow streamline.

Solution:

Find relation between and using the mass conservation

Since we choose the streamline as the control volume (*CV*), there is no mass flow across it. For the *CV*, is the unit normal vector and it is assumed that the *CV* has a unit depth.

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |

Thus,

|  |  |
| --- | --- |
|  | (3) |

Momentum conservation

The pressure is uniform and so there is no net pressure force. The flow is assumed incompressible and steady, so the momentum conservation equation applies only across the sections 1 and 3 without any unsteady terms.

|  |  |
| --- | --- |
|  | (4) |
|  | (5) |
|  | (6) |

Substitute from the mass conservation equation (3) into (6),

|  |  |
| --- | --- |
|  | (7) |

Thus,

|  |  |
| --- | --- |
|  | (8) |

Or, in a non-dimensional form

|  |  |
| --- | --- |
|  | (9) |

where,

 ; ;

and is the chord length of the airfoil.

Method 2: Rectangular control volume



Fig. 2 Rectangular control volume

Solution:

Mass conservation

Now, there is outflow of mass (and -momentum) across the sections 2 and 4. Then, the mass conservation equation () is written as,

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

Momentum conservation

The -momentum equation (4) can be expressed as

|  |  |
| --- | --- |
|  | (12) |

If it is assumed that the *x*-directional velocity at the sections 2 and 4 are nearly same as the free stream velocity, i.e. *u*(*x*) ≈ *U*∞, then, the second and fourth integrals at the left hand side of the equation (12) can be rewritten as

|  |  |
| --- | --- |
|  | (13) |

By using the relation (11) from the mass conservation, the right hand side of the equation (13) can be rewritten as

|  |  |
| --- | --- |
|  | (14) |

Then, the *x*-momentum equation (12) becomes as

|  |  |
| --- | --- |
|  | (15) |
|  | (16) |

Thus,

|  |  |
| --- | --- |
|  | (17) |

Or, in a non-dimensional form

|  |  |
| --- | --- |
|  | (18) |

where,

 ; ;

Example

 = 14.4 m/s, = 0.3048 m, AOA = 16°

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
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|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *i* | *yi* (m) | *ui* (m/s) | *yi*\* | *ui*\* |
| 1 | 0.200 | 14.4384 | 0.65617 | 1.00000 |
| 2 | 0.100 | 13.9520 | 0.32808 | 0.96631 |
| 3 | 0.050 | 13.1723 | 0.16404 | 0.91231 |
| 4 | 0.025 | 12.8620 | 0.08202 | 0.89082 |
| 5 | 0.015 | 12.8298 | 0.04921 | 0.88859 |
| 6 | 0.010 | 12.2982 | 0.03281 | 0.85178 |
| 7 | 0.005 | 10.5453 | 0.01640 | 0.73037 |
| 8 | 0.003 | 9.4002 | 0.00984 | 0.65106 |
| 9 | 0.000 | 7.9273 | 0.00000 | 0.54904 |
| 10 | -0.003 | 6.6970 | -0.00984 | 0.46383 |
| 11 | -0.005 | 8.3346 | -0.01640 | 0.57725 |
| 12 | -0.008 | 10.9333 | -0.02625 | 0.75724 |
| 13 | -0.010 | 13.0791 | -0.03281 | 0.90586 |
| 14 | -0.015 | 13.2519 | -0.04921 | 0.91783 |
| 15 | -0.025 | 13.1977 | -0.08202 | 0.91407 |
| 16 | -0.050 | 13.3596 | -0.16404 | 0.92529 |
| 17 | -0.100 | 13.5565 | -0.32808 | 0.93892 |
| 18 | -0.151 | 13.6128 | -0.49541 | 0.94282 |

 |

Pitot measured velocity profile (left) and the measurement data (right), where *y*\* = 0 is at the trailing edge (TE) of the wing model, and the measurement is at about one inch behind the TE.

Drag coefficient using the momentum integral method

The integration may be evaluated numerically such that

where,



Comparisons of the drag coefficient