## 57:020 Mechanics of Fluids and Transfer Processes Exercise Notes for the Pipe Flow TM

# Measurement of Flow Rate, Velocity Profile and Friction Factor in Pipe Flows S. Ghosh, M. Muste, M. Wilson, S. Breczinski, and F. Stern

## 1. Purpose

The purpose of this investigation is to provide students with *hands-on* experience using a pipe stand test facility and modern measurement systems including pressure transducers, pitot probes, and computerized data acquisition with Labview software, to measure flow rate, velocity profiles, and friction factors in smooth and rough pipes, determining measurement uncertainties, and comparing results with benchmark data. Additionally, this laboratory will provide an introduction to PIV analysis, using an ePIV system with a step-up model.

#### 2. Experimental Design

## 2.1 Part 1: Pipe Flow

The experiments are conducted in an instructional airflow pipe facility (Figure 1). The air is blown into a large reservoir located at the upstream end of the system. Pressure builds up in the reservoir, forcing the air to flow through any of the three horizontal pipes. Pressure taps are located on each pipe, at intervals of 1.524m, for static pressure measurements. Characteristics for each of the pipes are provided in Appendix A. At the downstream end of the system, the air is directed downward and back, through any of three pipes of varying diameters fitted with Venturi meters (Figure 2). The top three valves control flow through the experimental pipes, while the bottom three valves control the Venturi meter to be used. The Venturi meter with 5.08cm diameter is used to measure the total flow rate, while the other two are kept closed. Six gate valves are used for directing the flow. The top and bottom 5.08cm pipes are used for measurements, while the middle one is kept closed during the experiment. Velocity measurements in the top and bottom pipes are obtained using pitot probe (Figure 3).

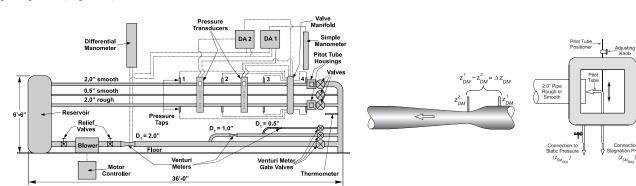


Figure 1. Airflow pipe system

Figure 2. Venturimeter

Figure 3. Pitot-probe

Pressures are acquired either manually, using simple and differential manometers for data acquisition, or automatically, with the manometers connected to an automated Data Acquisition (DA) system that converts pressure to voltages using pressure transducers. Data acquisition is controlled and interfaced by Labview software, described in Appendix B. The schematic of the two alternative measurement systems is provided in Figure 4.

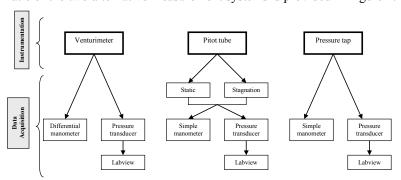


Figure 4. Manual and automated measurement systems used in the experiment

All pressure taps on the pipes, Venturi meters, and pitot probes have 0.635cm diameter quick coupler connections that can be hooked up to the pressure transducers.

### 2.1.1 Data reduction (DR) equations

In fully developed, axisymmetric pipe flow, the axial velocity u = u(r), at a radial distance r from the pipe centerline, is independent of the direction in which r is measured (Figure 5). However, the shape of the velocity profile is different for laminar and turbulent flows.

Laminar and turbulent flow regimes are distinguished by the flow Reynolds number, defined as

$$Re = \frac{VD}{V} = \frac{4Q}{\pi DV}$$
 (1)

Where, V is the average pipe velocity, D is the pipe diameter, Q is the pipe flow rate, and v is the kinematic viscosity of the fluid. For fully developed laminar flow (Re < 2000), an analytical solution for the differential equations of the fluid flow (Navier-Stokes and continuity) can be obtained. For turbulent pipe flows (Re > 2000), there is no exact solution, hence semi-empirical laws for velocity distribution are used instead.

The pipe head loss due to friction is obtained from the Darcy-Weisbach equation:

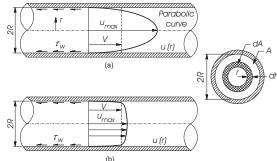


Figure 5. Velocity distributions for fully developed pipe flow: a) laminar flow; b) turbulent flow

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{2}$$

where, f is the (Darcy) friction factor, L is the length of the pipe over which the loss occurs,  $h_f$  is the head loss due to viscous effects, and g is the gravitational acceleration. The Moody diagram provides the friction factor for pipe flows with smooth and rough walls in laminar and turbulent regimes. The friction factor depends on the Reynolds number and the relative roughness k/D of the pipe (for large enough Re, the friction factor is solely dependent on the relative roughness).

Velocity distributions in the pipes are measured with Pitot tubes housed in glass-walled boxes (Figure 3). The data reduction equation (DRE) for the measurement of the velocity profiles is obtained by applying Bernoulli's equation for the Pitot tube:

$$u(r) = \left[ \frac{2 \cdot g \rho_{w}}{\rho_{a}} \cdot \left[ z_{SM_{Stat}}(r) - z_{SM_{Stat}} \right] \right]^{1/2}$$
(3)

where u(r) is the velocity at the radial position r, g is the gravitational acceleration,  $z_{SM_{Stor}}(r)$  is the stagnation pressure

head determined by the Pitot probe located at radial position r, and  $z_{SM_{Stat}}$  is the static pressure head in the pipe, equal to that of the ambient pressure inside the glass-walled box. These pressure head readings are given in height of a liquid column (ft of water). The DRE for the friction factor is one of the Darcy Weisbach equation forms (Roberson & Crowe, 1997), given as follows:

$$f = \frac{g\pi^2 D^5}{8LQ^2} \frac{\rho_w}{\rho_a} \left( z_{SM_i} - z_{SM_j} \right)$$
 (4)

where  $\rho_w$ , is the density of water,  $\rho_a$  is the density of air, L is the pipe length between pressure taps i and j, and  $z_{SMi} - z_{SMj}$  is the difference in pressure between pressure taps i and j. The flow rate Q is directly measured using the calibration equations for the Venturi meters (Rouse, 1978):

$$Q = C_d A_t \sqrt{2g\Delta z_{DM} \cdot \frac{\rho_w}{\rho_a}}$$
 (5)

where  $C_d$  is the discharge coefficient,  $A_t$  is the contraction area, and  $\Delta z_{DM}$  is the head drop across the Venturi, measured in height of a liquid column (ft of water) by the differential manometer or the pressure transducer. Appendix A lists Venturi meter characteristics. Alternatively, the flow rate can be determined by integrating the measured velocity distribution over the pipe cross-section, as follows:

$$Q_i = 2\pi \int_0^r u(r)rdr \tag{6}$$

#### 2.2 Part 2: ePIV

EFD Lab 1 investigated the use of ePIV as a method for visualizing streamlines around a circular cylinder. This laboratory will further explore the uses of Particle Image Velocimetry (PIV) to track fluid motion and calculate velocity vectors to describe the flow around a step-up model.

In ePIV analysis, a seeded fluid is illuminated by a laser sheet, and a camera takes rapid photographs of the fluid flow, at a rate of 30 Hz. Four parameters are used to control the camera settings;

- Brightness This controls the overall brightness of the image. For the best PIV results, brightness should be set to a medium-low value.
- Exposure This controls how long the camera sensors are exposed per image frame taken. Higher values correspond to shorter exposure times, and lower values correspond to longer exposure times. PIV analysis benefits from high exposure values (short exposure times), to facilitate software tracking of patterns of particles.
- Gain This controls the sensitivity of the sensors per unit time. Using higher gain will amplify the signal obtained by the sensors, so typically higher gain values are needed for images taken with short exposure times, which would otherwise be very dark. However, increasing the gain has a side effect: using higher gain increases the noise in the image.
- Frames This specifies how many images the camera will take, for PIV analysis. At least two images are needed to process vectors, and taking more will allow the software to average results and reduce precision error.

After images are captured, they are processed to determine velocity vectors and magnitudes. The software takes a pair of consecutive images and breaks it into many small regions, called interrogation windows. In each interrogation window, the PIV software compares the two images, determines how far the pattern of particles has moved in the amount of time between the two images, and calculates a single velocity vector for that window. This is repeated across the entire measurement area, generating a vector field. With the ePIV system, three PIV parameters can be adjusted.

- Window Size This sets the size (in pixels) of the interrogation window. Ideally, smaller windows are desired, because they show more flow detail, averaging over a smaller region of the flow. However, if values are too small, fewer particles pass through the interrogation window, which can result in unstable vector computation.
- Shift Size This determines the distance (in pixels) that the software moves to start a new interrogation window. For example, if a window size of 80 and a shift size of 40 were used, the software would compute a vector in the first 80x80 interrogation window, and then shift 40 pixels, computing a second vector in a new 80x80 window. The two windows would overlap by 50%. A smaller shift size results in more vectors being computed, but the increased overlap means that some of the data reported is repeated between the vectors.
- PIV Pairs This specifies how many pairs of images are used for PIV calculations. PIV analysis compares any two consecutive images, if 10 images are captured, up to 9 PIV pairs can be specified for computation. Results computed for each individual pair are averaged together, reducing precision error.

## 3. Experimental Process

## 3.1 Part 1: Pipe Flow

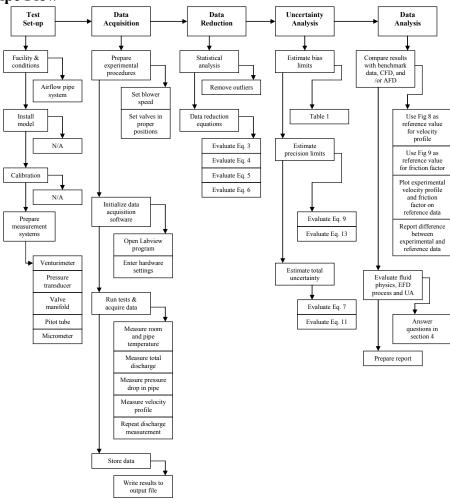


Figure 6. EFD Process

## 3.1.1 Test-setup

The experimental measurement systems for the manual and automated configurations are shown below:

Manual Data Acquisition	Automated Data Acquisition	
Facility (Figure 1)	Facility (Figure 1)	
Thermometers (room and inside the setup)	Thermometers (room and inside the pipe)	
Venturi meter (Figure 2)	Venturi meter (Figure 2)	
Pitot-tube assembly (Figure 3)	Pitot-tube assembly (Figure 3)	
Micrometer for Pitot positioning (Figure 3)	Micrometer for Pitot positioning (Figure 3)	
Simple manometer	DA (see Appendix B)	
Differential manometer	DA (see Appendix B)	
DA manifold	DA manifold	

## 3.1.2 Data Acquisition

Each student group will obtain velocity distributions and will determine the friction factor for one of the 5.08cm pipes, either the top (smooth) or bottom (rough) pipe. Data acquired with the DA are recorded electronically and will subsequently be used for data reduction, using the **Data reduction sheet**. The experimental procedure follows the sequence described below, and is the same for both rough and smooth pipes:

- 1. Starting with the low velocity initially set, gradually increase the flow rate until the desired *Re* (96,000) is attained in the test section. The desired *Re* can be achieved for both upper and lower pipes, with a setting of 35% on the blower motor controller and control valves fully open. Make at least five flow rate measurements with the 5.08cm-contraction-diameter venturi meter. The other two venture meters should be kept closed. The blower should be set to 15% for the first measurement and incremented by 5% for each additional measurement, up to 35% for the last of the five measurements. Record your readings from both the manometer and pressure transducer. Use Labview to record the venture meter reading. The remainder of the experiment will be carried out at 35% blower settings.
- 2. Take temperature readings with the digital thermometer (resolution 0.1 °F) of the ambient air and the inside of the pipe, for calculating the corresponding water and air densities, respectively. Input the temperature readings as requested by DA software interface. Since the temperature will increase during the experiment, take three temperature readings, at the beginning, in the middle, and at the end of the measurements.
- 3. The velocity distribution is obtained by using the DA to measure stagnation heads across the full pipe diameter, along with the readings of the static heads, using the appropriate Pitot-tube assembly. Measure stagnation heads at radial intervals no greater than 5 mm. The recommended spacing for half of the diameter of the upper and lower pipes is 0, 5, 10, 15, 20, 23, and 24 mm. Pitot tube position within the pipe is measured with a micrometer (resolution of 0.01 mm). To establish precision limits for velocity profiles, measurements closest to the pipe wall (at 24mm) should be taken 10 times.
- 4. Keeping the blower setting at 35%, measure the pressure heads at taps 1, 2, 3, and 4 sequentially as indicated in Figure 1, using the DA, by connecting each tap to the pressure transducer. To establish precision limits for the friction factor, measurements at taps 3 and 4 should be repeated 10 times. The repeated measurements should be made alternatively between taps 3 and 4. It is important to note that the pressure in the pipe system fluctuates when opening or closing manifold valves, hence it is necessary to wait a few seconds between consecutive measurements for the pressure fluctuation to settle down.

#### 3.1.3 Data Reduction

Data reduction for pipe flow includes the following steps:

- 1. Use the average temperatures  $T_w$  and  $T_a$  to determine  $\rho_w$ ,  $\rho_a$ , and  $\nu_a$  from fluid property tables. Determine the flow rate Q in the individual pipes using Equation (6), and the corresponding Re using Equation (1). The method for calculating flow rate (Equation 6) in individual pipes is explained in the **Data reduction sheet.**
- 2. Compare the flow rate readings taken with the manometer and pressure transducer.
- 3. Calculate velocity distribution profiles for the pipe that you tested, using Equation (3). Plot the measured velocity profile, including the total velocity uncertainties calculated for measurements at the centerline and near the wall. Compare the measured velocity distribution with the benchmark data provided in Figure 8.
- 4. Calculate the friction factor f for the pipe that you tested, using Equation (4). Use pressure readings from taps 3 and 4, where the flow is fully developed. Compare f with benchmark data, including the uncertainty band for the measured f.

#### 3.1.4 Uncertainty Analysis

Uncertainties for the experimentally-measured velocities and friction factor will be evaluated. The methodology for estimating uncertainties follows the AIAA S-071 Standard (AIAA, 1995) as summarized in Stern et al. (1999), for multiple tests (M = 10). Figure 7 shows two block diagrams depicting error propagation methodology for velocity and friction factor. Elemental errors for each of the measured independent variables in the data reduction equations should be identified using the best available information for bias errors, and using repeated measurements for precision errors. In this analysis, we will consider only the largest bias limits, and we will neglect correlated bias errors. The spreadsheet for evaluating the uncertainties is provided in **Data reduction sheet**. The spreadsheet includes bias limit estimates for the individual measured variables.

#### UA for Velocity Profile

The DRE for the velocity profile, Equation (3), is of the form:  $u(r) = F(g, \rho_w, \rho_a, z_{SM \, stag}, z_{SM \, stag})$ . We will only consider bias limits for  $z_{SM \, stag}$  and  $z_{SM \, stag}$ . The total uncertainty for velocity measurements is:

$$U_u^2 = B_u^2 + P_u^2 (7)$$

The bias limit,  $B_u$ , and the precision limit,  $P_u$ , for velocities are given by:

$$B_u^2 = \sum_{i=1}^{J} \theta_i^2 B_i^2 = \theta_{Z_{SM \, stag}}^2 B_{Z_{SM \, stag}}^2 + \theta_{Z_{SM \, stat}}^2 B_{Z_{SM \, stat}}^2$$
 (8)

$$P_u = KS_u / \sqrt{M} \tag{9}$$

where  $S_u$  is the standard deviation of the repeated velocity measurements. K = 2 for (M =) 10 repeated measurements. The sensitivity coefficients  $\theta$  (calculated using mean values for the independent variables) are:

$$\theta_{Z_{SM \, stag}} = \frac{1}{\left(z_{SM \, stag} - z_{SM \, stat}\right)^{0.5}} \left(0.5g \, \frac{\rho_{w}}{\rho_{a}}\right)^{0.5} \, \left(s^{-1}\right), \quad \theta_{Z_{SM \, stat}} = \frac{-1}{\left(z_{SM \, stag} - z_{SM \, stat}\right)^{0.5}} \left(0.5g \, \frac{\rho_{w}}{\rho_{a}}\right)^{0.5} \, \left(s^{-1}\right) \, (10)$$

#### **UA for Friction Factor**

The DRE for the friction factor, Equation (4), is of the form:  $f = F(g, D, L, Q, \rho_w, \rho_a, z_{SMi}, z_{SMj})$ . We will only consider bias limits for  $z_{SMi}$  and  $z_{SMj}$ . The total uncertainty for the friction factor is:

$$U_f^2 = B_f^2 + P_f^2 (11)$$

The bias limit,  $B_f$ , and the precision limit,  $P_f$ , for the result are given by:

$$B_f^2 = \sum_{i=1}^J \theta_i^2 B_i^2 = \theta_{z_{SMi}}^2 B_{z_{SMi}}^2 + \theta_{z_{SMj}}^2 B_{z_{SMj}}^2$$
 (12)

$$P_f = KS_f / \sqrt{M} \tag{13}$$

where  $S_f$  is the standard deviation of the repeated friction factor measurements. K = 2 for (M =) 10 repeated measurements. The sensitivity coefficients  $\theta$  (calculated using mean values for the independent variables) are:

$$\theta_{Z_{SMI}} = \frac{g\pi^2 D^5}{8LQ^2} \frac{\rho_w}{\rho_a} (1) \quad (m^{-1}) \qquad \theta_{Z_{SMI}} = \frac{g\pi^2 D^5}{8LQ^2} \frac{\rho_w}{\rho_a} (-1) \quad (m^{-1})$$
 (14)

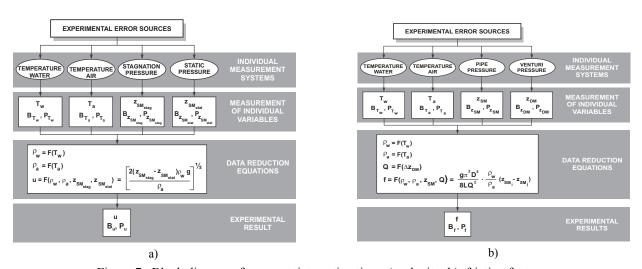


Figure 7. Block diagrams for uncertainty estimation: a) velocity; b) friction factor

Table 1. Bias limits for the individual variables included in the data reduction equations

Variable	Bias Limit		
	Estimation based on:	Value	
$B_{\mathbf{z}_{\mathfrak{M}i}},B_{\mathbf{z}_{\mathfrak{M}j}}\left(\mathbf{m}\right)$	Last significant digit	0.0003048	
$B_{\mathbf{Z}_{\mathrm{SM}\mathrm{stagnation}}}, B_{\mathbf{Z}_{\mathrm{SM}\mathrm{static}}}(\mathbf{m})$	Last significant digit	0.0003048	

#### 3.1.5 Data Analysis

Measurements obtained in the experiments will be compared with benchmark data. The benchmark data for velocity distribution is provided in numerical and graphical form in Figure 8. The benchmark data for friction factor is provided by the Moody diagram (Figure 9) and by the Colebrook-White-based formula (Roberson and Crowe, 1997):

$$f = \frac{0.25}{\left[\log\left(\frac{k/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2}$$
 (15)

The following questions relate to fluid physics, the EFD process, and uncertainty analysis. The solutions to these questions **must be included** in the Data analysis section of the lab report. Use the **Data reduction sheet** and attach it to your lab report.

- 1. Comment on the differences in flow rate readings obtained by the manometer and pressure transducer.
- 2. Plot the velocity profile u(r) obtained from the experiment, normalized by the maximum velocity in the pipe  $(u/U_{max})$ , against radial distance r, normalized by maximum radius (r/R). Plot the Schlichting data given in Figure 8 on the same plot. Compare the two profiles. Choose a point near the wall where the value of r/R is close to 1. Show the total percentage of uncertainty at that point using an uncertainty band.
- 3. Plot the head (in ft of water) at each pressure tap as a function of distance along the pipe. Comment on the pressure head drop distribution along the pipe and comment on uncertainties and unaccounted error sources.
- 4. Calculate the friction factor and compare your results with the Moody diagram. Show the experimental value of the friction factor on the Moody diagram, along with the uncertainty band.
- 5. What is the advantage of using non-dimensional forms for variables, such as those shown in Figures (8) and (9)?

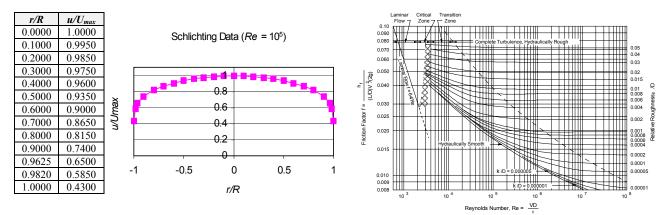


Figure 8. Benchmark data for the velocity profile

Figure 9. Benchmark data (Moody chart) for pipe friction factor

## 3.2 Part 2: ePIV

#### 3.2.1 Test Setup

Prior to the experiment, your TA will set up the ePIV system with a Step-up flow insert.

#### 3.2.2 Data Acquisition

The ePIV experimental procedure follows the steps listed below:

- 1. Turn on the ePIV system by flipping the switch on the back of the device.
- 2. Adjust the knob on the front of the ePIV system to maximize the flow rate.
- 3. On the computer desktop, open the "FLOWEX" software. Click on the "Acquire" button, on the left side of the screen.
- 4. Specify camera parameters for PIV image acquisition. Recommended values are 35, 100, 100, and 10, for brightness, exposure, gain, and frames, respectively.
- 5. Click on the "Capture" button to acquire 10 images. When FLOWEX returns to the "Acquire" dialogue, press F5 to refresh the screen and view your images, which should appear similar to the example in Figure 10, below. If your images look significantly different, modify your camera settings and re-capture the images.

- 6. Once you have satisfactory results, click on the "Analyze" button on the left side of the FLOWEX screen.
- 7. Specify parameters for PIV processing. Values of 80, 20, and 9 are recommended for window size, shift size, and PIV pairs, respectively.
- 8. Click on the "Process" button to begin PIV processing. This computation will take a few minutes to complete. When FLOWEX returns to the "Analyze" dialogue, press F5 to refresh the screen to view your results. You should be able to see images of a velocity vector field and velocity magnitude contours. Scroll the screen down below the two images under the "Results" section. Right-click on "Velocity vector field data" and save the file to a working directory on the computer. Rename the file to include your group number

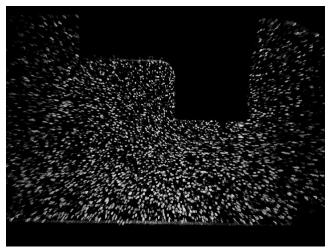


Figure 10. Sample raw data for ePIV step-up insert

#### 3.2.3 Data Reduction

Data reduction for ePIV includes the following steps:

- 1. Open the text file containing your velocity vector data and copy the entire contents of the file. Open the file "EFD2\_ePIV\_Data\_Reduction," and paste the copied data into the green cells on the first tab, labeled "Raw Velocity Vector Data."
- 2. The second tab in the Excel file, labeled "Calculations," lays out the x-components of your velocity vector data into a matrix corresponding to the geometry of the step-up ePIV model, and calculates the average velocity and flow rate for every x-value in the recorded data.
- 3. Plot the calculated average velocities and flow rates versus x-position.

#### 3.2.4. Data Analysis

Your lab report should include your plots of average velocity and flowrate versus x-position. You should answer the following questions and include them as well:

- 1. What happens to the average flow velocity as the cross-sectional area of the channel narrows? Why does this happen?
- 2. How does the flow rate change with x-position? Is this expected? Why or why not?

#### 4. References

Roberson, J.A. and Crowe, C.T. (1997). *Engineering Fluid Mechanics, 7th* edition, Houghton Mifflin, Boston, MA. Schlichting, H. (1968). *Boundary-Layer Theory*, McGraw-Hill, New York, NY.

Rouse, H. (1978). Elementary Mechanics of Fluids, Dover Publications, Inc., New Yoirk, NY.

Stern, F., Muste, M., Beninati, L-M, Eichinger, B. (1999). "Summary of Experimental Uncertainty Assessment Methodology with Example," IIHR Report No. 406, Iowa Institute of Hydraulic Research, The University of Iowa, Iowa City, IA.

## APPENDIX A

## SPECIFICATIONS FOR THE EXPERIMENTAL FACILITY COMPONENTS

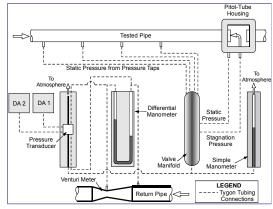
Table A1. Pipe characteristics

Experimental Pipe	Тор	Middle	Middle Bottom	
Diameter (mm)	52.38	25.4	52.93	
Internal Surface	Smooth, $k = 0.025 \text{ mm}$	Smooth	Rough, $k = 0.04 \text{ mm}$	
Number of Pressure Taps	4	8	4	
Tap Spacing (ft)	5	2.5	5	

Table A2. Venturi meter characteristics

Venturi specifications	Small	Medium	Large
Contraction Diameter, $D_t$ (mm)	12.7	25.4	51.054
Discharge Coefficient, $C_d$	0.915	0.937	0.935





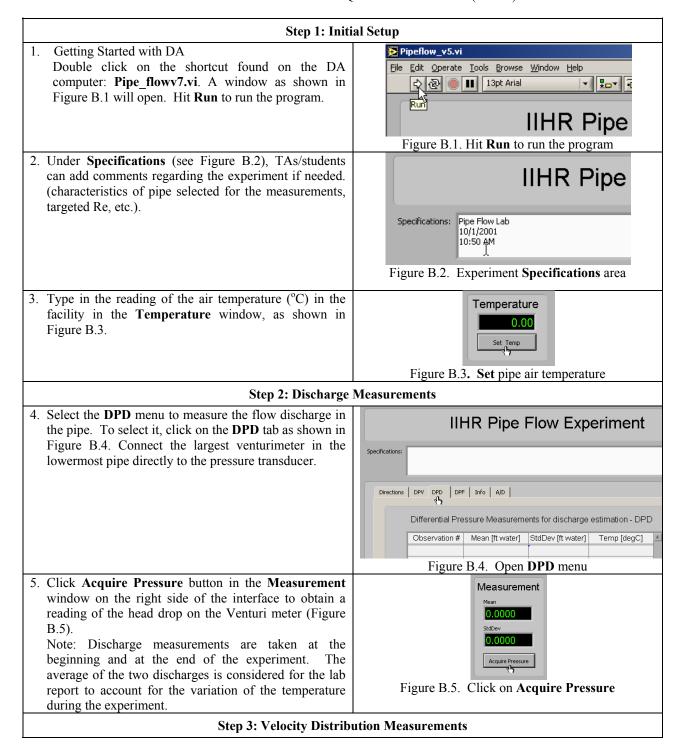
a) Photograph of experimental setup

b) Schematic of experimental setup

Figure A.1. Layout of the data acquisition systems

#### APPENDIX B

#### THE AUTOMATED DATA ACQUISITION SYSTEM (ADAS)



Velocity data will be measured with the appropriate pitot-tube according to the instructions given by the TA. Select the **DPV** tab, see Figure B.6. Connect the stagnation point on the pitot probe to the high side of the transducer and leave the low side open.

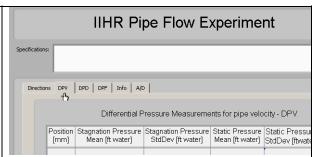


Figure B.6. Click on **DPV** tap to measure **D**ifferential Pressure for Velocity

- 6. Move the Pitot tube in the housing at the desired location for the velocity measurement (e.g. 20 mm from the centerline). Click Acquire Pressure (Figure B.7). The screen shown in Figure B.7 will then prompt the user for the pitot-tube location. Enter Pitot-tube position in the dialog box. Click **OK** to start the measurement.
- Set valves for Stagnation Pressure When ready, enter position (in mm) and press OK: 12 Figure B.7. Enter position of pitot-tube

Set valves for Static Pressure

When ready, press OK:

7. Following step 7, the screen shown in Figure B.8 will appear. Open the stagnation point and connect the static point from the pitot probe to the high side of the shown in Figure B.8.

transducer, in this case also the low side of the transducer remains open. Click OK on the screen Note: To establish precision limits for the simple manometer measurements, measurements should be taken at least 10 times. The repeated measurements should be made using an alternative pattern to avoid

successive measurements at the same location. Velocities are displayed graphically in a window after

Figure B.8. Click **OK** when ready for static pressure measurement

each measurement is taken. 8. Record final ambient and pipe air temperatures as indicated in step 3.

## **Step 4: Friction Factor Measurements**

9. Select **DPF** tab in the main menu (Figure B.9). Choose the desired pressure tap that is to be measured and connect it to the high side of the pressure transducer and leave the low side open to atmosphere.

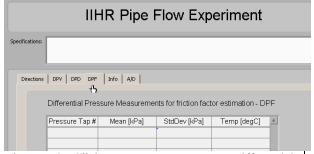


Figure B.9. Click on **DPF** tap to measure **D**ifferential Pressure for Friction Factor

10. Then enter the pressure tap number in the window shown in Figure B.10. Click OK. Click on Acquire **Pressure** as shown at Step 7 to make the measurement. Close the finger valve on the manifold and open the valve leading to the next measurement location.

Note: The pressure drop along the pipe is shown on a plot and ideally a linear curve should be observed.



Figure B.10. Enter 1 for tap  $Z_{sm1}$ , 2 for tap  $Z_{sm2}$  ...etc.

