Review for Exam3

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Chapter 8 Flow in Conduits
Pipe Flow: Laminar vs. Turbulent

• Reynolds number

\[ Re = \frac{\rho V D}{\mu} \]

- \( Re < Re_{crit} \approx 2,000 \)
- \( Re_{crit} < Re < Re_{trans} \)
- \( Re > Re_{trans} \approx 4,000 \)
Entrance Region and Fully Developed

- Entrance Length, $L_e$:
  - Laminar flow: $Le/D = 0.06Re$ \( (L_{e,\text{max}} = 0.06Re_{\text{crit}} \sim 138D) \)
  - Turbulent flow: $Le/D = 4.4Re^{1/6}$ \( (20D < Le < 30D \text{ for } 10^4 < Re < 10^5) \)
Pressure Drop and Shear Stress

- Pressure drop, $\Delta p = p_1 - p_2$, is needed to overcome viscous shear stress.
- The nature of shear stress is strongly dependent on whether the flow is laminar or turbulent.
- Friction factor (or Darcy friction factor)

$$f = \frac{8\tau_w}{\rho V^2}$$
Fully-developed Laminar Flow

• Exact solution, \( u(r) = V_c \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \)

• Wall shear stress

\[
\tau_w = -\mu \left( \frac{du}{dr} \right)_{r=R} = \frac{8\mu V}{D}
\]

Where, \( V = \frac{Q}{A} \)

• Friction factor,

\[
f = \frac{8\tau_w}{\rho V^2} = \frac{64}{\rho DV \mu} = \frac{64}{Re}
\]
Fully-developed Turbulent Flow

\[ \tau_w = f(D, V, \mu, \rho, \varepsilon) \]

\[ \rightarrow k - r = 6 - 3 = 3 \ \Pi 's \]

\[ f = \frac{8\tau_w}{\rho V^2} ; \quad Re = \frac{\rho V D}{\mu} ; \quad \text{Roughness} = \frac{\varepsilon}{D} \]

\[ \therefore f = \phi \left( Re, \frac{\varepsilon}{D} \right) \]
Moody Chart

\[ f = \frac{\rho V D}{\mu} \]

[Diagram of Moody Chart with labels for laminar flow, transition range, and wholly turbulent flow.]
Moody Chart – Contd.

• Colebrook equation

\[ \frac{1}{\sqrt{f}} = -2 \log \left( \frac{\varepsilon/D}{3.7} + \frac{2.51}{Re \sqrt{f}} \right) \]

• Haaland equation

\[ \frac{1}{\sqrt{f}} = -1.8 \log \left[ \left( \frac{\varepsilon/D}{3.7} \right)^{1.1} + \frac{6.9}{Re} \right] \]
Major Loss and Minor Losses

- **Energy equation**
  \[
  \frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L
  \]

- **Head loss**: \( h_L = h_f + h_m \)
  - Major loss due to friction:
    \[ h_f = f \frac{L V^2}{D 2g} \]  \( \text{ (Darcy – Weisbach equation)} \)
  - Minor loss due to pipe system components
    \[ h_m = \sum K_L \frac{V^2}{2g} \]  \( (K_L: \text{Loss coefficient}) \)
Pipe Flow Examples

• Type I: Determine head loss $h_L$ (or pressure drop)

• Type II: Determine flow rate $Q$ (or the average velocity $V$)

• Type III: Determine pipe diameter $D$

• For types II and III, iteration process is needed
Type I Problem

• Typically, $V$ and $D$ are given $\rightarrow Re$ and $\varepsilon / D$

\[
f = \phi \left( \frac{\rho V D}{\mu}, \frac{\varepsilon}{D} \right)
\]

\[
h_L = f \frac{L V^2}{D 2g}
\]
Type II Problem

• $Q$ (thus $V$) is unknown $\rightarrow Re$?
• Solve energy equation for $V = \text{function}(f)$, for example

$$h_p = \frac{V^2}{2g} + f \frac{L}{D} \frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

Or

$$V = \sqrt{\frac{2gh_p}{1 + f \frac{L}{D} + \sum K_L}}$$

Guess $f \rightarrow V \rightarrow Re \rightarrow f_{new}$; Repeat until $f$ is converged
Type III Problem

• $D$ is unknown $\rightarrow Re$ and $\varepsilon/D$?
• Solve energy equation for $D = \text{function}(f)$, for example,

$$
D = \left[ \frac{8LQ^2}{\pi^2 g h_f} \right]^{\frac{1}{5}} \cdot f^{\frac{1}{5}}
$$

Guess $f \rightarrow D \rightarrow Re$ and $\varepsilon/D \rightarrow f_{new}$; Repeat until $f$ is converged
Chapter 9 Flow over Immersed Bodies
Fluid Flow Categories

• **Internal flow**: Bounded by walls or fluid interfaces
  – Ex) Duct/pipe (Ch. 8), turbo machinery, open channel/river

• **External flow**: Unbounded or partially bounded. Viscous and inviscid flow regions
  – Ex) Flow around vehicles and structures
    • **Boundary layer flow**: High Reynolds number flow around streamlined bodies without flow separation
    • **Bluff body flow**: Flow around bluff bodies with flow separation

• **Free shear flow**: Absence of walls
  – Ex) Jets, wakes, mixing layers
Basic Considerations

- Drag, $D$: Resultant force in the direction of the upstream velocity
  \[ C_D = \frac{D}{\frac{1}{2} \rho V^2 A} = \frac{1}{\frac{1}{2} \rho V^2 A} \left( \int_S (p - p_\infty) n \cdot \hat{i} dA + \int_S \tau_w t \cdot \hat{i} dA \right) \]
  \[ \left\{ \begin{array}{l} t/\ell \ll 1 \quad C_f \gg C_Dp \quad \text{Streamlined body} \\
                 t/\ell \sim 1 \quad C_Dp \gg C_f \quad \text{Bluff body} \end{array} \right. \]
  where, $t$ is the thickness and $\ell$ the length of the body

- Lift, $L$: Resultant force normal to the upstream velocity
  \[ C_L = \frac{L}{\frac{1}{2} \rho V^2 A} = \frac{1}{\frac{1}{2} \rho V^2 A} \left( \int_S (p - p_\infty) j dA \right) \]
Boundary Layer

- Boundary layer theory assumes that viscous effects are confined to a thin layer, $\delta$
- There is a dominant flow direction (e.g., $x$) such that $u \sim U$ and $v \ll u$
- Gradients across $\delta$ are very large in order to satisfy the no-slip condition; thus, $\frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}$

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}
\]
Laminar boundary layer

- Prandtl/Blasius solution

\[ u = U_\infty f'(\eta) \]

\[ v = \frac{1}{2} \sqrt{\frac{\nu U_\infty}{x}} (\eta f' - f) \]

\[ \tau_w = 0.332 U^{3/2} \frac{\sqrt{\rho \mu}}{\sqrt{x}} \]

\[ \frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} ; \ c_f = \frac{0.664}{\sqrt{Re_x}} ; \ C_f = \frac{1.328}{\sqrt{Re_L}} \]
Turbulent boundary layer

- \( \frac{u}{U} \approx \left( \frac{y}{\delta} \right)^{\frac{1}{7}} \) one-seventh-power law
- \( c_f \approx 0.02 Re_{\delta}^{-\frac{1}{6}} \) power-law fit
- \( \frac{\delta}{x} = \frac{0.16}{Re_x^{\frac{7}{1}}} ; c_f = \frac{0.027}{Re_x^{\frac{7}{1}}} ; C_f = \frac{0.031}{Re_L^{\frac{7}{1}}} \)

- Valid for a fully turbulent flow over a smooth flat plate from the leading edge.
- Better results for sufficiently large \( Re_L \)
Turbulent boundary layer – Contd.

• Alternate forms by using an experimentally determined shear stress formula:

\[ \tau_w = 0.0225 \rho U^2 \left( \frac{\nu}{U\delta} \right)^{\frac{1}{4}} \]

\[ \frac{\delta}{x} = 0.37 Re_x^{-\frac{1}{5}} ; \quad c_f = \frac{0.058}{Re_x^{\frac{1}{5}}} ; \quad C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} \]

• Valid only in the range of the experimental data; \( Re_L = 5 \times 10^5 \sim 10^7 \) for smooth flat plate
Turbulent boundary layer – Contd.

- Other empirical formulas for smooth flat plates ("tripped" by some roughness or leading edge disturbance to make the flow turbulent from the leading edge):

\[ \frac{\delta}{L} = c_f (0.98 \log Re_L - 0.732) \]

\[ c_f = (2 \log Re_x - 0.65)^{-2.3} \]

\[ C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} \]
Turbulent boundary layer – Contd.

• Composite formulas (for flows initially laminar and subsequently turbulent with $Re_t = 5 \times 10^5$):

\[
C_f = \frac{0.031}{Re_l^{\frac{1}{7}}} - \frac{1440}{Re_L}
\]

\[
C_f = \frac{0.074}{Re_l^{\frac{1}{5}}} - \frac{1700}{Re_L}
\]

\[
C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1700}{Re_L}
\]
Turbulent boundary layer – Contd.

\[ C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} \]

Turbulent smooth plate

\[ C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} \quad (5 \times 10^5 < Re_L < 10^7) \]

Transitional

\[ C_f = \frac{0.455}{(\log_{10} Re_L)^{2.58}} - \frac{1700}{Re_L} \]

\[ C_f = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1700}{Re_L} \quad (5 \times 10^5 < Re_L < 10^7) \]

Laminar

\[ C_f = \frac{1.328}{\sqrt{Re_L}} \]

\[ C_f = \frac{0.031}{Re_L^{\frac{1}{5}}} - \frac{1440}{Re_L} \]
Bluff Body Drag

• In general,

\[ D = f(V, L, \rho, \mu, c, t, \varepsilon, \ldots) \]

• Drag coefficient:

\[ C_D = \frac{D}{\frac{1}{2} \rho V^2 A} = \phi \left( AR, \frac{t}{L}, Re, \frac{c}{V}, \frac{\varepsilon}{L}, \ldots \right) \]

• For bluff bodies experimental data are used to determine \( C_D \)
Shape dependence

• The blunter the body, the larger the drag coefficient

• The amount of streamlining can have a considerable effect
Reynolds number dependence

- **Very low $Re$ flow ($Re < 1$)**
  - Inertia effects are negligible (creeping flow)
  - $C_D \sim Re^{-1}$
  - Streamlining can actually increase the drag (an increase in the area and shear force)

- **Moderate $Re$ flow ($10^3 < Re < 10^5$)**
  - For streamlined bodies, $C_D \sim Re^{-1/2}$
  - For blunt bodies, $C_D \sim$ constant

- **Very large $Re$ flow (turbulent boundary layer)**
  - For streamlined bodies, $C_D$ increases
  - For relatively blunt bodies, $C_D$ decreases when the flow becomes turbulent ($10^5 < Re < 10^6$)

- For extremely blunt bodies, $C_D \sim$ constant
Separation

- Fluid stream detaches from a surface of a body at sufficiently high velocities.
- Only appears in viscous flows.
- Inside a separation region: low-pressure, existence of recirculating/backflows; viscous and rotational effects are the most significant.
Surface roughness

- For streamlined bodies, the drag increases with increasing surface roughness.
- For extremely blunt bodies, the drag is independent of the surface roughness.
- For blunt bodies, an increase in surface roughness can actually cause a decrease in the drag.
Lift

\[ C_L = \frac{L}{\frac{1}{2} \rho U^2 A} \]

\[ L = C_L \cdot \frac{1}{2} \rho U^2 A \]
Magnus Effect

- Lift generation by spinning
- Breaking the symmetry causes a lift

(a) Potential flow over a stationary cylinder
(b) Potential flow over a rotating cylinder
Minimum Flight Velocity

• Total weight of an aircraft should be equal to the lift

\[ W = F_L = \frac{1}{2} C_{L,max} \rho V_{min}^2 A \]

Thus,

\[ V_{min} = \sqrt{\frac{2W}{\rho C_{L,max} A}} \]