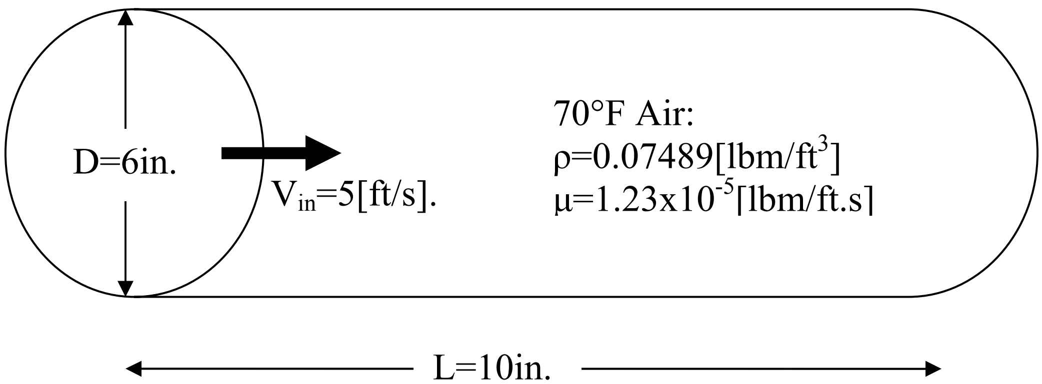
Example problem: Displacement thickness

A small, axisymmetric, low-speed wind tunnel is built to calibrate hot wires. The diameter of the test section is 6.0[inch], and its length is 10.0[inch]. The air is at 70°F. At a uniform air speed of 5.0[ft/s] at the test section inlet, by how much will the centerline air speed accelerate by the end of the test section? What should the engineers do to eliminate this acceleration?

Solution:



At the end of the test section, local Reynolds number is computed as;

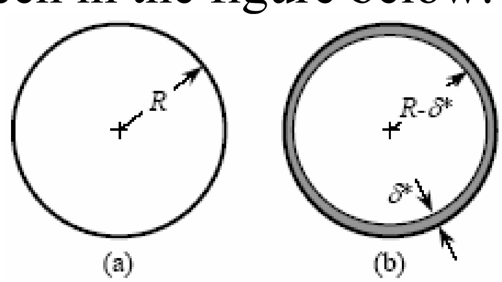
$$\mathrm{Re}\,x\Big|_{End} = \frac{\rho V L}{\mu} = 25359 < \text{Critical Reynolds nnumber} = 1\times 10^5$$

Therefore, the flow is laminar for all regions.

Use displacement thickness formula for laminar flow as;

$$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}} \rightarrow \delta^* = \frac{1.72}{\sqrt{\mathrm{Re}_x}} \times L \cong 9\times 10^{-3}[ft]$$

We can think that the diameter at exit becomes narrow by δ^* seen in the figure below:



Volumetric flow rate must be continuous between inlet and exit:

$$A_{in}V_{in} = A_{out}V_{out} \rightarrow V_{out} = \frac{A_{in}}{A_{out}}V_{in}$$

$$\therefore V_{out} = \frac{A_{in}}{A_{out}} V_{in} = \frac{\left(0.5 \big/ 2\right)^2}{\left(0.5 - 9 \times 10^{-3} \big/ 2\right)^2} \times 5.0 = 5.18[ft/s]$$

$$\rightarrow \frac{V_{out} - V_{in}}{V_{in}} \times 100 = 3.7\%$$

Engineers need to make the exit wider by $9x10^{-3}$[ft] in order to get rid of this acceleration, or they can put suctions along the wall in order to get rid of the increasing of displacement thickness.