Water is pumped between two reservoirs at a flow rate $Q = 0.2 \text{ ft}^3/\text{s}$ through a pipe with a total length $\ell = 400 \text{ ft}$ and a diameter $d = 2 \text{ in}$. and several minor losses. The roughness ratio is $\varepsilon/d = 0.001$. Compute the pump horsepower, $P$, required.

\begin{equation}
P = \rho g Q h_p \; ; \; \rho = 1.94 \text{ slugs/ft}^3; \; \nu = 0.000011 \text{ ft}^2/\text{s}^2; \; g = 32.2 \text{ ft/s}^2; \; 1 \text{ hp} = 550 \text{ ft-lbf/s}
\end{equation}

\begin{itemize}
  \item **Energy Eq.:**
  \begin{equation}
  \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{V^2}{2g} \left( \frac{f \ell}{d} + \sum K_L \right)
  \end{equation}

  \item **Friction factor, $f$:**
  \begin{equation}
  \frac{1}{\sqrt{f}} = 1.8 \log \left[ \left( \frac{\varepsilon}{d} \right)^{1.11} + \frac{6.9}{R_e} \right]
  \end{equation}
\end{itemize}

**Solution:**

Since $p_1 = p_2$ and $V_1 = V_2 \approx 0$, the energy equation becomes

\begin{equation}
\begin{aligned}
h_p &= z_2 - z_1 + \frac{V^2}{2g} \left( \frac{f \ell}{d} + \sum K_L \right) \\
(\text{+3 points})
\end{aligned}
\end{equation}

With the flow rate known,

\begin{equation}
V = \frac{Q}{A} = \frac{0.2 \text{ ft}^3/\text{s}}{\frac{1}{4} \pi \left( \frac{2}{12} \text{ ft} \right)^2} = 9.17 \text{ ft/s}
\end{equation}

Calculate the Reynolds number,

\begin{equation}
Re = \frac{V d}{\nu} = \frac{9.17 \left( \frac{2}{12} \text{ ft} \right)}{0.000011} = 139,000
\end{equation}

\begin{tabular}{|c|c|}
\hline
Loss & $K_L$ \\
\hline
Sharp entrance & 0.5 \\
Open globe valve & 6.9 \\
12-in bend & 0.25 \\
Regular 90° elbow & 0.95 \\
Half-closed gate valve & 2.7 \\
Sharp exit & 1.0 \\
\hline
\end{tabular}
For $\varepsilon/d = 0.001$, the pipe friction factor,

\[ f = -1.8 \log \left( \frac{\varepsilon/d}{3.7} + \frac{6.9}{Re} \right) = 0.0214 \]  

(\text{+3 points})

Minor losses are

\[ \sum K_L = 0.5 + 6.9 + 0.25 + 0.95 + 2.7 + 1.0 = 12.3 \]  

(\text{+2 points})

Thus, the pump head $h_p$ becomes,

\[ h_p = 100 \text{ ft} + \frac{(9.17 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} \left( \frac{0.0214(400)}{2} + 12.3 \right) = 183 \text{ ft} \]

The pump must provide a power to the water of

\[ P = \rho g Q h_p = [1.94(32.2) \text{ lbf/ft}^3](0.2 \text{ ft}^3/s)(183 \text{ ft}) = 2286 \text{ ft} \cdot \text{lbf/s} \]

The conversion factor is 1 hp = 550 ft·lbf/s. Therefore

\[ P = \frac{2286}{550} = 4.2 \text{ hp} \]  

(\text{+2 points})