## Interest Formulas
## Chapter 4

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## Single Payment Compound Interest

- **P**= (P)resent sum of money
- **i**= (i)nterest per time period (usually years)
- **n**= (n)umber of time periods (usually years)
- **F**= (F)uture sum of money that is equivalent to **P** given an interest rate **i** for **n** periods
- $F=P(1+i)^n$ F=P(F/P,i,n)
- Single Payment Present Worth Formula
  - $P=F(1+i)^{-n}$ P=F(P/F,i,n)



## Steps to solution

- Step 1: Identify cash flow (P and F)
- Step 2: Identify interest rate (i) and number of periods
- Step 3: Select appropriate table or formula
  - $F=P(1+i)^n$ $P=F(1+i)^{-n}$
  - F=P(F/P,i,n) P=F(P/F,i,n)
- Step 4: Perform calculation
- All four steps are a small part of an actual engineering decision



## Key points

- Time value of money, $1,000 today is not the same as $1,000 one hundred years from now
- Equivalence provides a common language for comparing present and future sums of money
- Equivalence depends on the assumed interest rate
- Notation for single payment compound interest:

  F=P(F/P,i,n) P=F(P/F,i,n)



## More Interest Formulae: Uniform Series A

- Uniform amount A at end of time period
- Uniform series = aggregation of several present values (P)
- $F=A(1+i)^{n-1}+\dots A(1+i)^2 + A(1+i)$
- Superposition principle - Lego building
  - See p 98 - 99 for derivation



## Uniform Series F/A A/F

1. Uniform Series Compound Amount Factor

(F/A,i,n) $[(1 + i)^n - 1]/i = F/A$

2. Uniform Series Sinking Fund Factor

(A/F,i,n) $i/[(1 + i)^n - 1] = A/F$

## Uniform Series Compound Amount F/A

- Determines future value (F) of periodic contributions (A)
- Example: Value of IRA given periodic contributions
- F=A(F/A,i,n)

Chapter 4 - 7

## Retirement in 25 years?

- Deposit $10,000 each year for 25 years
- Interest rate is 15%, compounded annually
- At the end of 25 years how much will you have for retirement?

Chapter 4 - 8

## Uniform Series Sinking Fund A/F

- Determines contribution/payment given a future value
- Example: Periodic contribution to IRA that is required to achieve goal
- A=F(A/F,i,n)

Chapter 4 - 9

## Uniform Series Capital Recovery A/P

- Determines contribution/payment given a present value
- Example: Income from an IRA that is possible given savings; loan repayment
- A=P(A/P,i,n)

Chapter 4 - 10

## A/P and P/A

<u>Uniform Series Capital Recovery Factor</u>

(Simple interest)

(A/P,i,n) $[i(1 + i)^n]/[(1 + i)^n - 1] = A/P$

Note: Inverse is Uniform Series Present Worth Factor P/A

Chapter 4 - 11

## Loan Repayment

- Car loan of $20,000
- Interest rate is 15%, compounded annually
- What are the annual repayments?

Chapter 4 - 12

## Examples

Examples 4-5, 4-6

Superposition principle can be used to modify cash flow descriptions to fit standard form.



## Steps to solving problems

- Identify variables (F, P, A, i, n)
- Draw diagram
- Convert to workable form
- Identify appropriate formula
- Perform calculations
- Verify against rough estimates



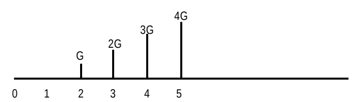
## Uniform series formulas covered thus far

- Uniform series compounded
  - F=A(F/A,i,n)
- Uniform series sinking fund
  - A=F(A/F,i,n)
- Uniform series capital recovery
  - A=P(A/P,i,n)
- Uniform series present worth value
  - P=A(P/A,i,n)



## Arithmetic Gradient

- graduated payments (G)
- A=G(A/G,i,n)      P=G(P/G,i,n)
- Example:  Increasing maintenance costs with aging equipment
- Note: G=0 at time =1






## Arithmetic Gradient

1. Arithmetic Gradient Present Worth Factor

(P/G,i,n)  $[(1 + i)^n - in - 1]/[i^2(1 + i)^n] = P/G$

2. Arithmetic Gradient Uniform Series

(A/G,i,n)  $[(1 + i)^n - in -1]/[i(1 + i)^n - I] = A/G$



## Geometric Gradient

- Determines uniform payments (A) given graduated payments (G) that increase at a constant percentage
- P=A(F/A,g,i,n)
- g=percent increase in A
- Two formulas, one for i=g and i<>g
- Unlike arithmetic  A starts at time 1
- Example:  IRA contributions increase with income

## Nominal and effective interest

- Nominal interest rate= Interest rate without consideration of compounding
- Effective interest rate= Nominal interest rate adjusted for compounding
- Nominal=Effective *IF* compounding period equals period of effective interest rate
- **Conversion to effective interest rate provides a basis to make comparisons**

Chapter 4 - 19

## Nominal and effective interest rates

- $i$ = Effective interest rate per interest period
- $r$ = Nominal interest rate per period
- $i_a$= Effective interest rate per year (annum)
- $i_s$ = Effective interest rate per sub period
- m= Number of compounding subperiods in the period used to define the nominal rate “r”

Chapter 4 - 20

## Nominal and effective interest rates

Effective interest rate, $i_P$, (period of compounding=period of interest) is used in formulas:

$i=i_P=(1+ i_s)^m-1$

$i=i_P=(1+r_P/m)^m-1$

$i_s$=interest per subperiod

$r_P$=nominal interest per period P

m=number of subperiods in period P

Nominal interest rate, $r_P=m \times i_s$

Continuous compounding: $i_a=e^r -1$

$F = P(1+ i_a)^n = P^* e^m$

Chapter 4 - 21

## Nominal interest rate of 12% compounded monthly

- What is the effective interest rate per month?
- What is the nominal interest rate per month?
- What is the effective interest rate per year?
- Does (F/A, 12%, 30) = (F/A, 1%, 360)?

Chapter 4 - 22

**Table 4-3 NOMINAL AND EFFECTIVE INTEREST RATES**

| *Nominal interest rate per year* | *Effective interest rate per year, $i_a$, when nominal rate is compounded* | | | | |
|---|---|---|---|---|---|
| ***r*** | ***Yearly*** | ***Semi-annually*** | ***Monthly*** | ***Daily*** | ***Continu-ously*** |
| 1% | 1.0000% | 1.0025% | 1.0046% | 1.0050% | 1.0050% |
| 2 | 2.0000 | 2.0100 | 2.0184 | 2.0201 | 2.0201 |
| 3 | 3.0000 | 3.0225 | 3.0416 | 3.0453 | 3.0455 |
| 4 | 4.0000 | 4.0400 | 4.0742 | 4.0809 | 4.0811 |
| 5 | 5.0000 | 5.0625 | 5.1162 | 5.1268 | 5.1271 |
| 6 | 6.0000 | 6.0900 | 6.1678 | 6.1831 | 6.1837 |
| 8 | 8.0000 | 8.1600 | 8.3000 | 8.3278 | 8.3287 |
| 10 | 10.000 | 10.2500 | 10.4713 | 10.5156 | 10.5171 |
| 15 | 15.000 | 15.5625 | 16.0755 | 16.1798 | 16.1834 |
| 25 | 25.000 | 26.5625 | 28.0732 | 28.3916 | 28.4025 |

Chapter 4 - 23

4-63 A student bought a guitar for $75 and agreed to pay $85 after 6 months. Nominal interest rate? Effective annual interest rate?

Chapter 4 - 24

## General problem-solving suggestions

- Draw the cash flow diagram
- Calculate a rough guess
  - Use a crude model: ignore interest, ignore compounding
  - Doubling rule: an amount doubles every 70/i% years
- Track units
  - Effective interest rate must have the same units for period of compounding as for period of interest
  - “n” must match “i” in tables

Chapter 4 - 25

## Overview of Chapter 4: Translation to common units

- Convert between values in future and present
- Convert between single values and series of values
- Convert between nominal interest rate and interest rate that considers effect of compounding (effective)
- Effective interest rate (period of compounding=period of interest) is used in formulas: $i=(1+i_s)^m-1$

  ($i_s$=interest per subperiod)

  (m= number of subperiods)

Chapter 4 - 26