Interest Formulas
Chapter 4

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Single Payment Compound Interest

P = (P)resent sum of money
i = (i)nterest per time period (usually years)
n = (n)umber of time periods (usually years)
F = (F)uture sum of money that is equivalent to P given an interest rate i for n periods
F = P(1+i)^n
F = P(F/P,i,n)
Single Payment Present Worth Formula
P = F(1+i)^{-n}
P = F(P/F,i,n)

Steps to solution

Step 1: Identify cash flow (P and F)
Step 2: Identify interest rate (i) and number of periods
Step 3: Select appropriate table or formula
   F = P(1+i)^n
   P = F(1+i)^{-n}
   F = P(F/P,i,n)
   P = F(P/F,i,n)
Step 4: Perform calculation
All four steps are a small part of an actual engineering decision

Key points

Time value of money, $1,000 today is not the same as $1,000 one hundred years from now
Equivalence provides a common language for comparing present and future sums of money
Equivalence depends on the assumed interest rate
Notation for single payment compound interest:
F = P(F/P,i,n)
P = F(P/F,i,n)

More Interest Formulæ: Uniform Series A

Uniform amount A at end of time period
Uniform series = aggregation of several present values (P)
F = A[1+i]^n + A(1+i)^{n-1} + ... + A(1+i)
Superposition principle - Lego building
See p 98 - 99 for derivation

Uniform Series F/A A/F

1. Uniform Series Compound Amount Factor
(F/A,i,n) = [(1 + i)^n - 1]/i = F/A

2. Uniform Series Sinking Fund Factor
(A/F,i,n) = i[(1 + i)^n - 1] = A/F
Uniform Series Compound Amount  \( F/A \)
- Determines future value (F) of periodic contributions (A)
- Example: Value of IRA given periodic contributions
- \( F = A(F/A,i,n) \)

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Retirement in 25 years?
- Deposit $10,000 each year for 25 years
- Interest rate is 15%, compounded annually
- At the end of 25 years how much will you have for retirement?

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Uniform Series Sinking Fund  \( A/F \)
- Determines contribution/payment given a future value
- Example: Periodic contribution to IRA that is required to achieve goal
- \( A = F(A/F,i,n) \)

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Uniform Series Capital Recovery  \( A/P \)
- Determines contribution/payment given a present value
- Example: Income from an IRA that is possible given savings; loan repayment
- \( A = P(A/P,i,n) \)

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A/P and P/A

Uniform Series Capital Recovery Factor
(Simple interest)

\[
(A/P,i,n) = \frac{i(1 + i)^n}{(1 + i)^n - 1} = A/P
\]

Note: Inverse is Uniform Series Present Worth Factor P/A

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Loan Repayment
- Car loan of $20,000
- Interest rate is 15%, compounded annually
- What are the annual repayments?

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Examples

Examples 4-5, 4-6

Superposition principle can be used to modify cash flow descriptions to fit standard form.

Steps to solving problems

- Identify variables (F, P, A, i, n)
- Draw diagram
- Convert to workable form
- Identify appropriate formula
- Perform calculations
- Verify against rough estimates

Uniform series formulas covered thus far

- Uniform series compounded
  \[ F = A(F/A, i, n) \]
- Uniform series sinking fund
  \[ A = F(A/F, i, n) \]
- Uniform series capital recovery
  \[ A = P(A/P, i, n) \]
- Uniform series present worth value
  \[ P = A(P/A, i, n) \]

Arithmetic Gradient

- Graduated payments (G)
  \[ A = G(A/G, i, n) \]
  \[ P = G(P/G, i, n) \]
- Example: Increasing maintenance costs with aging equipment
- Note: \( G = 0 \) at time =1

Geometric Gradient

- Determines uniform payments (A) given graduated payments (G) that increase at a constant percentage
  \[ P = A(F/A, g, i, n) \]
- \( g = \) percent increase in A
- Two formulas, one for \( i = g \) and \( i <> g \)
- Unlike arithmetic, A starts at time 1
- Example: IRA contributions increase with income
Nominal and effective interest

- Nominal interest rate = Interest rate without consideration of compounding
- Effective interest rate = Nominal interest rate adjusted for compounding
- Nominal = Effective IF compounding period equals period of effective interest rate
- Conversion to effective interest rate provides a basis to make comparisons

Nominal and effective interest rates

- $i$ = Effective interest rate per interest period
- $r$ = Nominal interest rate per period
- $i_a$ = Effective interest rate per year (annum)
- $i_s$ = Effective interest rate per sub period
- $m$ = Number of compounding subperiods in the period used to define the nominal rate $r$

Effective interest rate, $i_a$, (period of compounding = period of interest) is used in formulas:

$$i_a = (1 + i_s)^m - 1$$

Continuous compounding: $i_a = e^r - 1$

$F = P(1 + i_a)^n = P*e^{rn}$

Nominal interest rate of 12% compounded monthly

- What is the effective interest rate per month?
- What is the nominal interest rate per month?
- What is the effective interest rate per year?
- Does $(F/A, 12\%, 30) = (F/A, 1\%, 360)$?

Table 4-3 NOMINAL AND EFFECTIVE INTEREST RATES

<table>
<thead>
<tr>
<th>Nominal interest rate per year</th>
<th>Effective interest rate per year, $i_a$, when nominal rate is compounded</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Yearly</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>1%</td>
<td>1.0000%</td>
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<tr>
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</tr>
<tr>
<td>25</td>
<td>25.000%</td>
</tr>
</tbody>
</table>

4-63 A student bought a guitar for $75 and agreed to pay $85 after 6 months. Nominal interest rate? Effective annual interest rate?
### General problem-solving suggestions

- Draw the cash flow diagram
- Calculate a rough guess
- Use a crude model: ignore interest, ignore compounding
- Doubling rule: an amount doubles every $70/i\%$ years
- Track units
- Effective interest rate must have the same units for period of compounding as for period of interest
- “n” must match “i” in tables

### Overview of Chapter 4: Translation to common units

- Convert between values in future and present
- Convert between single values and series of values
- Convert between nominal interest rate and interest rate that considers effect of compounding (effective)
- Effective interest rate (period of compounding=period of interest) is used in formulas:  
  \[ i = (1+i_\text{s})^m - 1 \]
  
  ($i_\text{s}$=interest per subperiod)
  ($m$= number of subperiods)