A Simple Introduction to Embedded Control Systems (PID Control)

1

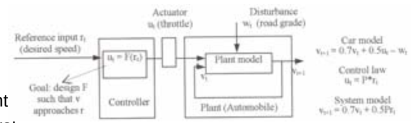
Acknowledgements

- The material in this lecture is adapted from:
  - F. Vahid and T. Givargis, **Embedded System Design—A Unified Hardware/Software Introduction**, John Wiley & Sons, 2002 (Chapter 9)
  - T. Wescott, “PID Without a PhD”
    - http://www.embedded.com/2000/0010/0010feat3.htm

2

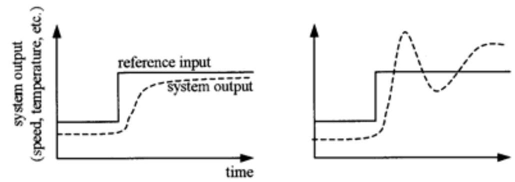
Control System

- Control physical system’s output
  - By setting physical system’s input
- Tracking
- E.g.
  - Cruise control
  - Thermostat cont
  - Disk drive control
  - Aircraft altitude control
  - Chemical processes
- Difficulty due to
  - Disturbance: wind, road, tire, brake; opening/closing door…
  - Human interface: feel good, feel right…



3

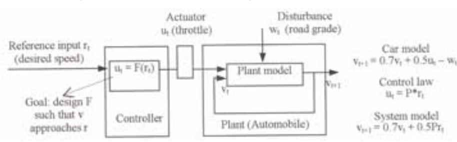
Tracking



4

Open-Loop Control Systems

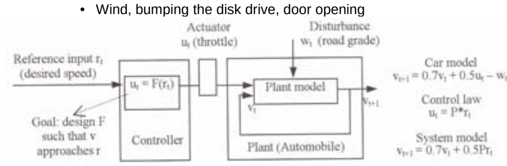
- Plant
  - Physical system to be controlled
    - Car, plane, disk, heater,…
- Actuator
  - Device to control the plant
    - Throttle, wing flap, disk motor,…
- Controller
  - Designed product to control the plant



5

Open-Loop Control Systems

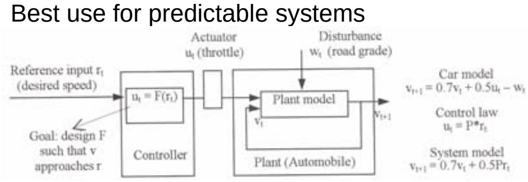
- Output
  - The aspect of the physical system we are interested in
    - Speed, disk location, temperature
- Reference
  - The value we want to see at output
    - Desired speed, desired location, desired temperature
- Disturbance
  - Uncontrollable input to the plant imposed by environment
    - Wind, bumping the disk drive, door opening



6

Other Characteristics of open loop

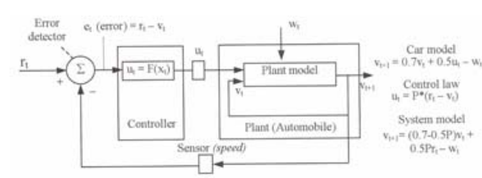
- Feed-forward control
- Delay in actual change of the output
- Controller doesn't know how well thing goes
- Simple
- Best use for predictable systems



7

Closed Loop Control Systems

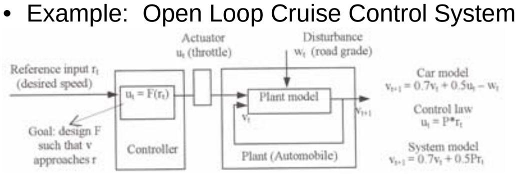
- Sensor
  - Measure the plant output
- Error detector
  - Detect Error
- Feedback control systems
- Minimize tracking error



8

Designing Open Loop Control System

- Develop a model of the plant
- Develop a controller
- Analyze the controller
- Consider Disturbance
- Determine Performance
- Example: Open Loop Cruise Control System



9

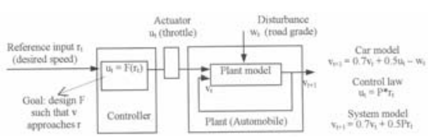
Model of the Plant

- May not be necessary
  - Can be done through experimenting and tuning
- But,
  - Can make it easier to design
  - May be useful for deriving the controller
- Example: throttle that goes from 0 to 45 degree
  - On flat surface at 50 mph, open the throttle to 40 degrees
  - Wait 1 “time unit”
  - Measure the speed, let's say 55 mph
  - Then the following equation satisfies the above scenario
    - $v_{t+1}=0.7*v_t+0.5*u_t$
    - 55 = 0.7*50+0.5*40
  - IF the equation holds for all other scenarios
    - Then we have a model of the plant

10

Designing the Controller

- Assuming we want to use a simple linear function
  - $u_t=F(r_t)= P * r_t$
  - r_t is the desired speed
- Linear proportional controller
- $v_{t+1}=0.7*v_t+0.5*u_t = 0.7*v_t+0.5P*r_t$
- Let $v_{t+1}=v_t$ at steady state = v_{ss}
- $v_{ss}=0.7*v_{ss}+0.5P*r_t$
- At steady state, we want $v_{ss}=r_t$
- P=0.6
  - I.e. $u_t=0.6*r_t$



Analyzing the Controller

- Let v_0=20mph, r_0=50mph
- $v_{t+1}=0.7*v_t+0.5(0.6)*r_t$
  $=0.7*v_t+0.3*50= 0.7*v_t+15$
- Throttle position is 0.6*50=30 degrees

Time (t)	v_t
0	20.00
1	29.00
2	35.30
3	39.71
4	42.80
5	44.96
6	46.47
7	47.53
8	48.27
9	48.79
10	49.15
11	49.41
12	49.58

12

Considering the Disturbance

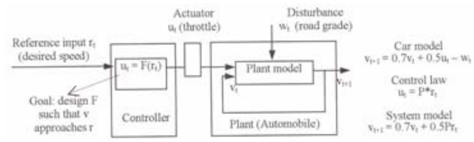
- Assume road grade can affect the speed
  - From –5mph to +5 mph
  - $v_{t+1}=0.7*v_t+10$
  - $v_{t+1}=0.7*v_t+20$

Time (t)	v_t	v_t for w = +5	v_t for w = –5
0	20.00	20.00	20.00
1	29.00	24.00	34.00
2	35.30	26.80	43.80
3	39.71	28.76	50.66
4	42.80	30.13	55.46
5	44.96	31.09	58.82
6	46.47	31.76	61.18
7	47.53	32.24	62.82
8	48.27	32.56	63.98
9	48.79	32.80	64.78
10	49.15	32.96	65.35
11	49.41	33.07	65.74
12	49.58	33.15	66.02

13

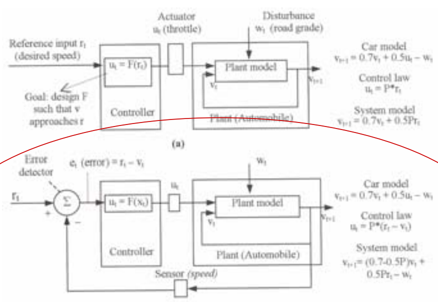
Determining Performance

- $V_{t+1}=0.7*v_t+0.5P*r_0-w_0$
- $v_1=0.7*v_0+0.5P*r_0-w_0$
- $v_2=0.7*(0.7*v_0+0.5P*r_0-w_0)+0.5P*r_0-w_0$
  $=0.7*0.7*v_0+(0.7+1.0)*0.5P*r_0-(0.7+1.0)w_0$
- $v_t=0.7^t*v_0+(0.7^{t-1}+0.7^{t-2}+\ldots+0.7+1.0)(0.5P*r_0-w_0)$
- Coefficient of v_t determines rate of decay of v_0
  - Bigger coefficient will result in slower response
  - >1 or <-1, v_t will grow without bound
  - <0, v_t will oscillate



14

Designing Closed Loop Control System



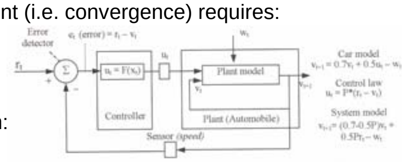
15

Stability

- $u_t = P*(r_t-v_t)$
- $v_{t+1} = 0.7v_t+0.5u_t-w_t = 0.7v_t+0.5P*(r_t-v_t)-w_t$
  $=(0.7-0.5P)*v_t+0.5P*r_t-w_t$
- $v_{t+1}=(0.7-0.5P)^t*v_0+((0.7-0.5P)^{t-1}+(0.7-0.5P)^{t-2}+\ldots+0.7$
  $0.5P+1.0)(0.5P*r_0-w_0)$

- Stability constraint (i.e. convergence) requires:

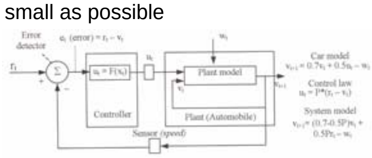
$|0.7-0.5P|<1$
$-1<0.7-0.5P<1$
$-0.6<P<3.4$
To avoid oscillation:
$(0.7 - 5 P) >= 0$
$P <=1.4$



16

Reducing effect of v_0

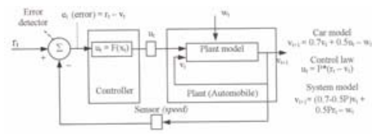
- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$
  $= (0.7-0.5P)*v_t + 0.5P*r_t - w_t$
- $v^t = (0.7-0.5P)^t * v_0 + ((0.7-0.5P)^{t-1} + (0.7-0.5P)^{t-2} + \ldots + 0.7-0.5P + 1.0)(0.5P*r_0 - w_0)$
- To reduce the effect of initial condition
  - 0.7-0.5P as small as possible
  - P=1.4



17

Avoid Oscillation

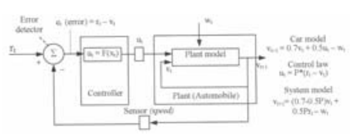
- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$
  $= (0.7-0.5P)*v_t + 0.5P*r_t - w_t$
- $v^t = (0.7-0.5P)^t * v_0 + ((0.7-0.5P)^{t-1} + (0.7-0.5P)^{t-2} + \ldots + 0.7-0.5P + 1.0)(0.5P*r_0 - w_0)$
- To avoid oscillation
  $0.7-0.5P >= 0$
  $P <= 1.4$



18

Perfect Tracking

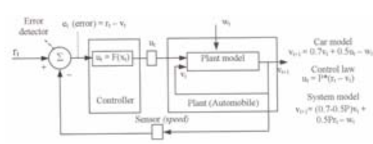
- $u_t = P * (r_t - v_t)$
- $v_{t+1} = 0.7v_t + 0.5u_t - w_t = 0.7v_t + 0.5P*(r_t - v_t) - w$
  $= (0.7-0.5P)*v_t + 0.5P*r_t - w_t$
- $v_{ss} = (0.7-0.5P)*v_{ss} + 0.5P*r_0 - w_0$
  $(1-0.7+0.5P)v_{ss} = 0.5P*r_0 - w_0$
  $v_{ss} = (0.5P/(0.3+0.5P)) * r_0 - (1.0/(0.3+0.5P)) * w_o$
- To make v_{ss} as close to r_0 as possible
  P should be as large as possible



19

Closed-Loop Design

- $u_t = P * (r_t - v_t)$
- Finally, setting P=3.3
  - Stable, track well, some oscillation
  $u_t = 3.3 * (r_t - v_t)$



20

Analyze the controller

- v_0=20 mph, r_0=50 mph, w=0
- $v_{t+1} = 0.7v_t+0.5P*(r_t-v_t)-w$
  $= 0.7v_t+0.5*3.3*(50-v_t)$
- $u_t = P * (r_t-v_t)$
  $= 3.3 * (50-v_t)$

- But valid u_t ranges from 0-45
- Controller saturates

Time	v_t	u_t
0	20.00	99.00
1	63.50	-44.55
2	22.18	91.82
3	61.43	-37.73
4	24.14	85.34
5	59.57	-31.58
6	25.91	79.50
7	57.89	-26.02
8	27.51	74.22
9	56.37	-21.01
10	28.95	69.46
...		
45	44.53	18.06
46	40.20	32.34
47	44.31	18.78
48	40.41	31.66
49	44.11	19.42
50	40.59	31.05
...		
ss	42.31	25.38

21

Analyze the controller

- v_0=20 mph, r_0=50 mph, w=0
- $v_{t+1} = 0.7v_t+0.5*u_t$
- $u_t = 3.3 * (50-v_t)$
  - Saturate at 0, 45
- Oscillation!
  - "feel bad"

Time	v_t	u_t	v_t	u_t
0	20.00	99.00	20.00	45.00
1	63.50	-44.55	36.50	44.55
2	22.18	91.82	47.83	7.18
3	61.43	-37.73	37.07	42.68
4	24.14	85.34	47.29	8.95
5	59.57	-31.58	37.58	40.99
6	25.91	79.50	46.80	10.55
7	57.89	-26.02	38.04	39.47
8	27.51	74.22	46.36	12.00
9	56.37	-21.01	38.45	38.10
10	28.95	69.46	45.97	13.31
...				
45	44.53	18.06	41.70	27.39
46	40.20	32.34	42.89	23.48
47	44.31	18.78	41.76	27.20
48	40.41	31.66	42.83	23.66
49	44.11	19.42	41.81	27.02
50	40.59	31.05	42.78	23.83
...				
ss	42.31	25.38	42.31	25.38

22

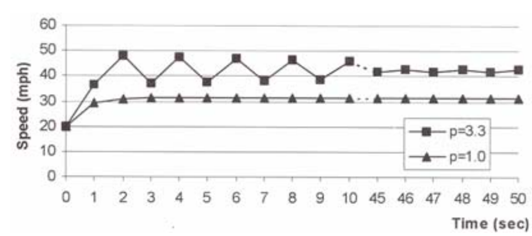
Analyze the controller

- Set P=1.0 to void oscillation
  - Terrible SS performance

Time	v_t	u_t	v_t	u_t	v_t	u_t
0	20.00	99.00	20.00	45.00	20.00	30.00
1	63.50	-44.55	36.50	44.55	29.00	21.00
2	22.18	91.82	47.83	7.18	30.80	19.20
3	61.43	-37.73	37.07	42.68	31.16	18.84
4	24.14	85.34	47.29	8.95	31.23	18.77
5	59.57	-31.58	37.58	40.99	31.25	18.75
6	25.91	79.50	46.80	10.55	31.25	18.75
7	57.89	-26.02	38.04	39.47	31.25	18.75
8	27.51	74.22	46.36	12.00	31.25	18.75
9	56.37	-21.01	38.45	38.10	31.25	18.75
10	28.95	69.46	45.97	13.31	31.25	18.75
...						
45	44.53	18.06	41.70	27.39	31.25	18.75
46	40.20	32.34	42.89	23.48	31.25	18.75
47	44.31	18.78	41.76	27.20	31.25	18.75
48	40.41	31.66	42.83	23.66	31.25	18.75
49	44.11	19.42	41.81	27.02	31.25	18.75
50	40.59	31.05	42.78	23.83	31.25	18.75
...						
ss	42.31	25.38	42.31	25.38	31.25	18.75

23

Analyzing the Controller



24

Minimize the effect of disturbance

- $v_{t+1} = 0.7v_t+0.5*3.3*(r_t-v_t)-w$
  - w=-5 or +5
- 39.74
  - Close to 42.31
  - Better than
    - 33
    - 66
- Cost
  - SS error
  - oscillation

Time	v_t	u_t	v_t	u_t
0	20.00	45.00	20.00	45.00
1	31.50	45.00	41.50	28.05
2	39.55	34.49	48.08	6.35
3	39.93	33.24	41.83	26.97
4	39.57	34.42	47.76	7.38
5	39.91	33.30	42.13	25.99
6	39.59	34.37	47.48	8.31
7	39.89	33.35	42.39	25.10
8	39.60	34.32	47.23	9.15
9	39.88	33.40	42.63	24.30
10	39.62	34.27	47.00	9.91
...				
45	39.76	33.78	44.52	18.09
46	39.72	33.91	45.21	15.82
47	39.76	33.78	44.55	17.97
48	39.73	33.91	45.17	15.92
49	39.76	33.79	44.58	17.87
50	39.73	33.90	45.14	16.02
...				
ss	39.74	33.85	44.87	16.92

25

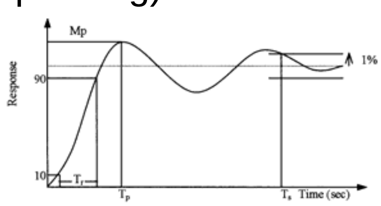
General Control System

- Objective
  - Causing output to track a reference even in the presence of
    - Measurement noise
    - Model error
    - Disturbances
- Metrics
  - Stability
    - Output remains bounded
  - Performance
    - How well an output tracks the reference
  - Disturbance rejection
  - Robustness
    - Ability to tolerate modeling error of the plant

26

Performance (generally speaking)

- Rise time
  - Time it takes from 10% to 90%
- Peak time
- Overshoot
  - Percentage by which Peak exceed final value
- Settling time
  - Time it takes to reach 1% of final value



27

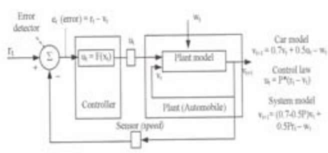
Plant modeling is difficult

- May need to be done first
- Plant is usually on continuous time
  - Not discrete time
    - E.g. car speed continuously react to throttle position, not at discrete interval
  - Sampling period must be chosen carefully
    - To make sure “nothing interesting” happen in between
    - I.e. small enough
- Plant is usually non-linear
  - E.g. shock absorber response may need to be 8^{th} order differential
- Iterative development of the plant model and controller
  - Have a plant model that is “good enough”

28

Controller Design: P

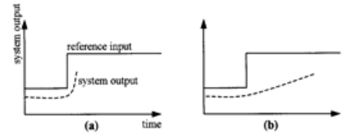
- Proportional controller
  - A controller that multiplies the tracking error by a constant
    - $u_t = P * (r_t - v_t)$
  - Closed loop model with a linear plant
    - E.g. $v_{t+1} = (0.7-0.5P)*v_t+0.5P*r_t-w_t$
- P affects
  - Transient response
    - Stability, oscillation
  - Steady state tracking
    - As large as possible
  - Disturbance rejection
    - As large as possible



29

Controller Design: PD

- Proportional and Derivative control
  - $u_t = P * (r_t-v_t) + D * ((r_t-v_t)-(r_{t-1}-v_{t-1})) = P * e_t+ D * (e_t-e_{t-1})$
- Consider the size of error over time
- Intuitively
  - Want to "push" more if the error is not reducing fast enough
  - Want to "push" less if the error is reducing really fast



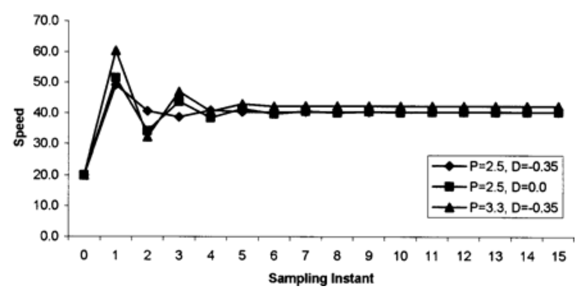
30

PD Controller

- Need to keep track of error derivative
- E.g. Cruise controller example
  - $v_{t+1} = 0.7v_t+0.5u_t-w_t$
  - Let $u_t = P * e_t + D * (e_t-e_{t-1})$, $e_t=r_t-v_t$
  - $v_{t+1}=0.7v_t+0.5*(P*(r_t-v_t)+D*((r_t-v_t)-(r_{t-1}-v_{t-1})))-w_t$
  - $v_{t+1}=(0.7-0.5*(P+D))*v_t+0.5D*v_{t-1}+0.5*(P+D)*r_t-0.5D*r_{t-1}-w_t$
  - Assume reference input and disturbance are constant, the steady-state speed is
    - $V_{ss}=(0.5P/(1-0.7+0.5P)) * r$
    - Does not depend on D!!!
- P can be set for best tracking and disturbance control
- Then D set to control oscillation/overshoot/rate of convergence

31

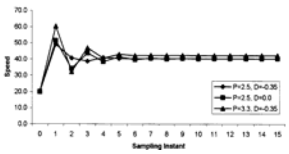
PD Control Example

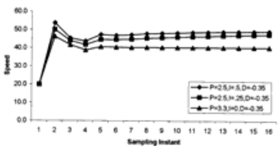


32

PI Control

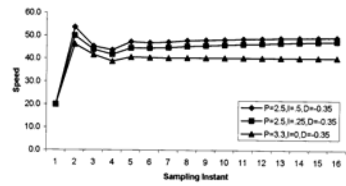
- Proportional plus integral control
  - $u_t = P*e_t + I*(e_0 + e_1 + \ldots + e_t)$
- Sum up error over time
  - Ensure reaching desired output, eventually
  - v_{ss} will not be reached until $e_{ss}=0$
- Use P to control disturbance
- Use I to ensure steady state convergence and convergence rate





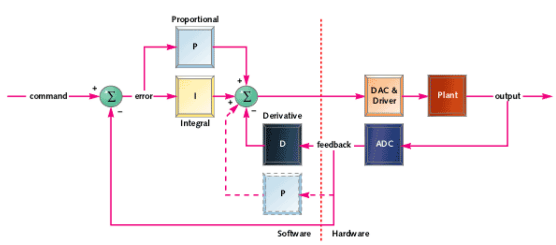
PID Controller

- Combine Proportional, integral, and derivative control
  - $u_t = P*e_t + I*(e_0 + e_1 + \ldots + e_t) + D*(e_t - e_{t-1})$
- Available off-the shelf



34

Block Diagram View of PID Controller



35

Software Coding

- Main function loops forever, during each iteration
  - Read plant output sensor
    - May require A2D
  - Read current desired reference input
  - Call PidUpdate, to determine actuator value
  - Set actuator value
    - May require D2A

```
void main()
{
    double sensor_value, actuator_value, error_current;
    PID_DATA pid_data;
    PidInitialize(&pid_data);
    while (1) {
        sensor_value = SensorGetValue();
        reference_value = ReferenceGetValue();
        actuator_value =
         PidUpdate(&pid_data,sensor_value,reference_value);
        ActuatorSetValue(actuator_value);
    }
```

36

Software Coding (continued)

- Pgain, Dgain, Igain are constants
- sensor_value_previous
  - For D control
- error_sum
  - For I control

```
typedef struct PID_DATA {
    double Pgain, Dgain, Igain;
    double sensor_value_previous; // find the derivative
    double error_sum; // cumulative error
}
```

37

Computation

- $u_t=P*e_t+I*(e_0+e_1+\ldots+e_t)+D*(e_t-e_{t-1})$

```
double PidUpdate(PID_DATA *pid_data, double sensor_value,
                 double reference_value)
{
    double Pterm, Iterm, Dterm;
    double error, difference;

    error = reference_value - sensor_value;
    Pterm = pid_data->Pgain * error; /* proportional term*/
    pid_data->error_sum += error; /* current + cumulative*/
    // the integral term
    Iterm = pid_data->Igain * pid_data->error_sum;
    difference = pid_data->sensor_value_previous -
                 sensor_value;
    // update for next iteration
    pid_data->sensor_value_previous = sensor_value;
    // the derivative term
    Dterm = pid_data->Dgain * difference;
    return (Pterm + Iterm + Dterm);
}
```

38

PID tuning

- Analytically deriving P, I, D may not be possible
  - E.g. plant not is not available, or to costly to obtain
- Ad hoc method for getting "reasonable" P, I, D
  - Start with a small P, I=D=0
  - Increase D, until seeing oscillation
    - Reduce D a bit
  - Increase P, until seeing oscillation
    - Reduce D a bit
  - Increase I, until seeing oscillation
- Iterate until can change anything without excessive oscillation

39

Excellent Reference for PID Control

- PID Without a PhD by Tim Wescott
- http://www.embedded.com/2000/0010/0010feat3.htm

40

Practical Issues with Computer-Based Control

- Quantization
- Overflow
- Aliasing
- Computation Delay

41

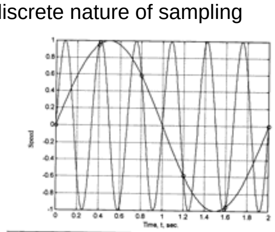
Quantization & Overflow

- Quantization
  - E.g. can't store 0.36 as 4-bit fractional number
  - Can only store 0.75, 0.59, 0.25, 0.00, -0.25, -050,-0.75, -1.00
  - Choose 0.25
    - Results in quantization error of 0.11
- Sources of quantization error
  - Operations, e.g. 0.50*0.25=0.125
    - Can use more bits until input/output to the environment/memory
  - A2D converters
- Overflow
  - Can't store 0.75+0.50 = 1.25 as 4-bit fractional number
- Solutions:
  - Use fix-point representation/operations carefully
    - Time-consuming
  - Use floating-point co-processor
    - Costly

42

Aliasing

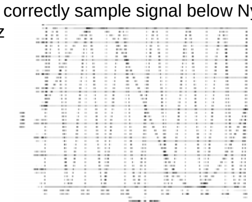
- Quantization/overflow
  - Due to discrete nature of computer data
- Aliasing
  - Due to discrete nature of sampling



43

Aliasing Example

- Sampling at 2.5 Hz, period of 0.4, the following are indistinguishable
  - y(t)=1.0*sin(6πt), frequency 3 Hz
  - y(t)=1.0*sin(πt), frequency of 0.5 Hz
- In fact, with sampling frequency of 2.5 Hz
  - Can only correctly sample signal below Nyquist frequency 2.5/2 = 1.25 Hz



44

Computation Delay

- Inherent delay in processing
  - Actuation occurs later than expected
- Need to characterize implementation delay to make sure it is negligible
- Hardware delay is usually easy to characterize
  - Synchronous design
- Software delay is harder to predict
  - Should organize code carefully so delay is predictable and minimized
  - Write software with predictable timing behavior (be like hardware)
    - Time Trigger Architecture
    - Synchronous Software Language and/or RTOS

45