z, Z and Z-R Relationships

Think Back

Radar reflectivity h for a distributed target, is defined by the following

$$s_t = V \sum_{Unit\ Volume} s_i = Vh$$

The units are $m^2\ m^{-3}$ or m^{-1} or cm^{-1}

$$P_r = \left(\frac{G^2 l^2 P_t q f c t}{1024 \ln(2) p^2} \right) \frac{h}{R^2} = C \frac{h}{R^2}$$

Radar equation for distributed targets

$$h = \frac{p^5 |k|^2}{l^4} z$$

Radar reflectivity

$$z = \sum_{Unit\ Volume} D^6$$

Radar reflectivity factor (mm^6/mm^3)

Combining these equations, we can write: $P_r = c_2 \frac{z}{R^2}$, and rearrange to get:

$$z = c_3 P_r R^2$$

c_3 is the so-called *radar constant*. It has units $mm^6/m^3\ mW^{-1}\ km^{-2}$. The radar reflectivity factor z has a tremendous dynamic range so it is convenient to express it on a decibel scale with a reference z = 1 mm^6/m^3

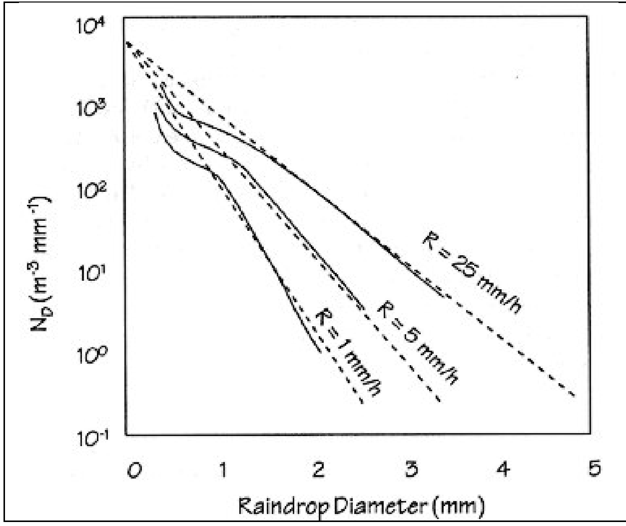
$$Z = 10 \log_{10} \left(\frac{z\ (mm^6/m^3)}{1\ (mm^6/m^3)} \right) \text{ with units dBZ}$$

Using this, we can write

$$z = c_3 P_r R^2$$
$$Z = C_3 + P_r + 20 \log_{10}(R)$$

where Z is measured in dBZ, C3 = $10\log_{10}(c_3)$, P_r is measured in dBm, and R is in km.

Marshall-Palmer DSD.



$$N_D = N_o e^{-lD}$$
$$l = 4.1R^{-0.21}$$

N_o = 8000 m^{-3} mm^{-1}
R = rainrate in mm/h
D = drop diameter in mm

Given a DSD one can compute a Z-R relationship. In practice, empirical relationships are used:

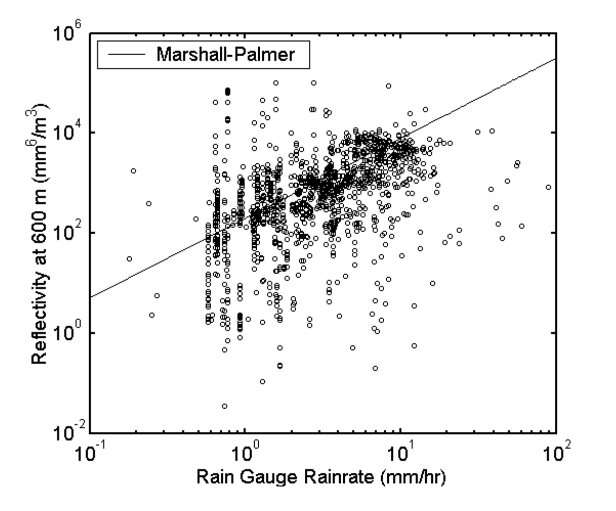
$$Z = AR^b$$
$$Z = 200R^{1.6} \text{ Marshall - Palmer or MP relationship}$$

There are many relationship in use. The table below is from the NOAA ROC:

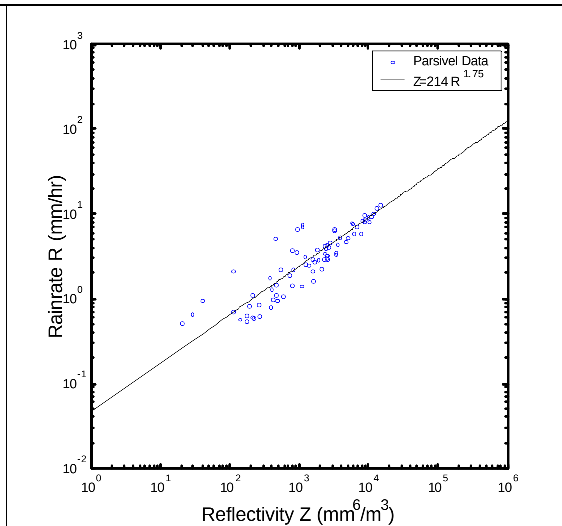
Table 1. Z-R RECOMMENDATIONS

RELATIONSHIP	Optimum for:	Also recommended for:
Marshall-Palmer ($z=200R^{1.6}$)	General stratiform precipitation	
East-Cool Stratiform ($z=130R^{2.0}$)	Winter stratiform precipitation - east of continental divide	Orographic rain - East
West-Cool Stratiform ($z=75R^{2.0}$)	Winter stratiform precipitation - west of continental divide	Orographic rain - West
WSR-88D Convective ($z=300R^{1.4}$)	Summer deep convection	Other non-tropical convection
Rosenfeld Tropical ($z=250R^{1.2}$)	Tropical convective systems	

Sample *Z-R* relationships measured in Iowa (note the X-Y axis are switched between the graphs).



Sample: 1-minute integration interval

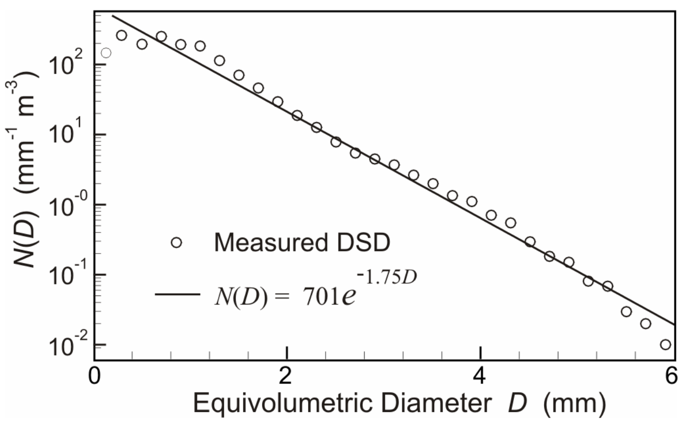


Sample: 5-minute integration interval.

Problem. The figure below shows a drop size distribution measured with a disdrometer. It also shows least squares fit to the data. Assume there are no drops outside of the range 0.2–6 mm.

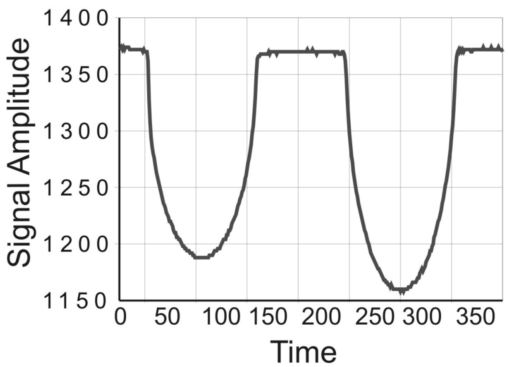
(a) Estimate the number of drops per unit volume (i.e., in one cubic meter).

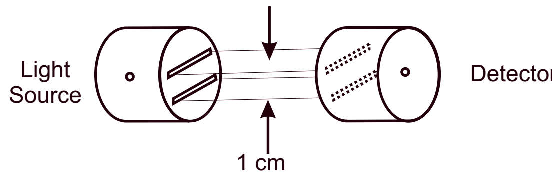
(b) Compute the radar reflectivity factor Z. Supply the proper units.



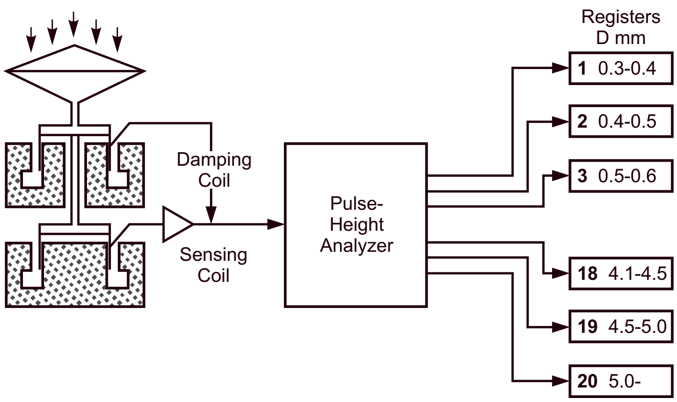
Disdrometers

Optical

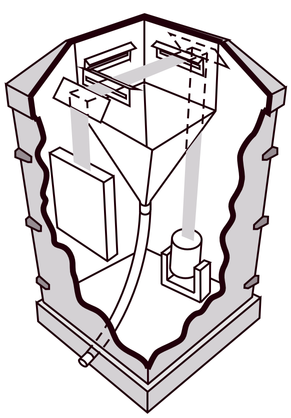




Impact (Joss-Waldvogel)



Video



Radar-type