Radar Equation for Point Targets

Consider an isotropic radiator transmitting a pulse with power P_t and a target that is at distance R. The target has an area A_σ. At a distance R from the transmitter the power density is

$$S = \frac{P_t}{4pR^2}$$

Next add an antenna with gain G that redirects the power towards the target. The power intercepted by the target with area A_σ is

$$P_s = \frac{GP_t A_s}{4pR^2}$$

Now make the assumption that (a) there are no losses at the target, and (b) the power intercepted by the target is reradiated isotropically. Some of the reradiated power reaches the transmitter. This amount is

$$P_r = \frac{P_s A_e}{4pR^2}$$

where A_e is the effective area of the transmitter antenna. Substitute the expression for P_s and we have

$$P_r = \frac{GP_t A_s}{4pR^2}\frac{A_e}{4pR^2} = \frac{GP_t A_s A_e}{(4p)^2 R^4}$$

The effective area of the antenna is by definition

$$A_e = \frac{G l^2}{4p}$$

Thus the received power is

$$P_r = \frac{GP_t A_s}{(4p)^2 R^4}\frac{G l^2}{4p} = \frac{G^2 l^2 P_t A_s}{64p^3 R^4}$$

The physical target area A_σ is related to, but different from what appears to the radar. Call the *radar cross sectional* (RCS) area s, and the received power is

$$\boxed{P_r = \frac{G^2 l^2 P_t s}{64p^3 R^4}}$$

Caveat: there are several variations of the radar equation in use. Some use energy rather than power, some include the radar pulse width, and different symbols are used. This does not change the basic relationships between transmitted power, distance from the radar, gain, etc.

This equation is called the

Example. The received power from a **point target** located at 110 km from a radar is 10^{-14} W. What is the received power of two identical point targets at 180 km?

Use the radar equation for point targets to determine the target area

$$P_r(R_1) = \frac{G^2 \lambda^2 P_t \sigma}{64\pi^3 R_1^4} \Rightarrow \sigma = P_r(R_1)\frac{64\pi^3 R_1^4}{G^2 \lambda^2 P_t} \text{ m}^2$$

where R_1 = 110 km. Use the radar equation for point targets to determine the received power at R_2 = 180 km:

$$\begin{aligned} P_r(R_2) &= \frac{G^2 \lambda^2 P_t \sigma}{64\pi^3 R_2^4} = \frac{G^2 \lambda^2 P_t}{64\pi^3 R_2^4} \cdot P_r(R_1)\frac{64\pi^3 R_1^4}{G^2 \lambda^2 P_t} \\ &= P_r(R_1)\frac{R_1^4}{R_2^4} \\ &= 10^{-14}\left(\frac{110}{180}\right)^4 \end{aligned}$$

This is the received power from one target. The received power from two point targets is twice this, or 2.78×10^{-15} W.

Radar Cross Section (RCS)

RCS and measured in m^2. It depends on three factors

$$\sigma = (\text{Projected Cross Section}) \times (\text{Reflectivity}) \times (\text{Directivity})$$

Target Geometric Cross Section (Projected Cross Section)

This is the geometric area/cross section *A* presented to the radar, i.e., the projection of the target. For example, the geometric area of a ship is larger broadside compared to head on. The units are m^2.

Target Reflectivity

Not all the intercepted power is reflected. For example, aircraft can be coated with absorbent materials, and there are inherent ohmic losses in all scatterers. Reflectivity is expressed as a ratio so it is dimensionless.

Target Directivity

This refers to the power scattered back in the direction of the transmitting radar. Simple, idealized targets scatter isotropically, but real targets don't. They scatter more in some directions than others. Target directivity is defined as the ratio of backscattered power to backscatter from a target that scatters isostropically. Directivity is dimensionless.

Sphere, radius = a	πa^2
Cylinder, radius = a, height = h	$\frac{2\pi a h^2}{\lambda}$
Flat plate, width = a, height = b	$\frac{4\pi a^2 b^2}{\lambda^2}$
Square trihedral corner reflector, width = length = height = a	$\frac{12\pi a^4}{\lambda^2}$

Some radar cross sections

The table above shows (the maximum) RCS for some simple geometric scatterers, under the assumption that the wavelength is much smaller that the largest dimension of the object, and that the range is much larger than the wavelength, and the reflectivity is 1. Note that for a sphere there is no dependence on wavelength.

Example. What is the maximum RCS for a flat plate 0.094 m $\times$ 0.094 m at 1 GHz and 10 GHz?

Solution

At 1 GHz, the wavelength is 0.1 m, so the RCS is

$$RCS_{1\,\mathrm{GHz}} = \frac{4\pi a^2 b^2}{\lambda^2} = \frac{4\pi \times 0.094^2 \times 0.094^2}{0.1^2} = 1\,\mathrm{m}^2$$

At 10 GHz, the wavelength is 10 times smaller, and the RCS is 100 times smaller, so

$$RCS_{10\,\mathrm{GHz}} = 0.01\,\mathrm{m}^2$$