Radar Equation for Point Targets

Consider an isotropic radiator transmitting a pulse with power P_t and a target that is at distance R. The target has an area A_σ. At a distance R from the transmitter the power density is

$$S = \frac{P_t}{4\pi R^2}$$

Next add an antenna with gain G that redirects the power towards the target. The power intercepted by the target with area A_σ is

$$P_\sigma = \frac{G P_t A_\sigma}{4\pi R^2}$$

Now make the assumption that (a) there are no losses at the target, and (b) the power intercepted by the target is reradiated isotropically. Some of the reradiated power reaches the transmitter. This amount is

$$P_r = \frac{P_\sigma A_e}{4\pi R^2}$$

where A_e is the effective area of the transmitter antenna. Substitute the expression for P_σ and we have

$$P_r = \frac{G P_t A_\sigma}{4\pi R^2} \frac{A_e}{4\pi R^2} = \frac{G P_t A_\sigma A_e}{(4\pi)^2 R^4}$$

The effective area of the antenna is by definition

$$A_e = \frac{G\lambda^2}{4\pi}$$

Thus the received power is

$$P_r = \frac{G P_t A_\sigma}{(4\pi)^2 R^4} \frac{G\lambda^2}{4\pi} = \frac{G^2 \lambda^2 P_t A_\sigma}{64\pi^3 R^4}$$

The physical target area A_σ is related to, but different from what appears to the radar. Call the *radar cross sectional* area σ, and the received power is

$$\boxed{P_r = \frac{G^2 \lambda^2 P_t \sigma}{64\pi^3 R^4}}$$

Caveat: there are several variations of the radar equation in use. Some use energy rather than power, some include the radar pulse width, and different symbols are used. This does not change the basic relationships between transmitted power, distance from the radar, gain, etc.