Review

Purpose of Simulation
- Purpose of simulation is insight, not numbers.
  - How a new or modified system will work?
  - Will it meet throughput expectations?
  - What happens to response time at peak periods?
  - Is the system resilient to short-term surges?
  - What is the effect of congestion and queuing?
  - What are the staffing requirements?
  - If problems occur, what is the cause and what is the remedy?
  - What is the system capacity?
  - What are the boundary conditions?

What is a System?
A system is a group of components, that mutually affects each others behavior, working together toward a common goal taking inputs and producing outputs.
You can describe a system by:
- Specifying the components the system consists of.
- Describing the characteristics of the components.
- And finally specifying the relations between these components.

Computer Simulation
- Computer simulation refers to methods for studying a wide variety of models of systems
  - Numerically evaluate the system on a computer
  - Use software to imitate the system's operations and characteristics, often over time
- Can be used to study simple models but should not use it if an analytical solution is available
- Real power of simulation is in studying complex models
- Simulation can tolerate complex models since we don’t even aspire to an analytical solution

Pieces of a Simulation Model
- Entities
  - Dynamic objects that move around, change status, affect and are affected by other entities
- Attributes
  - Characteristic of all entities: describes, differentiates. Example: Time of arrival, Due date, Priority, Cake
  - Attribute value is tied to a specific entity
- (Global) Variables
  - Reflects a characteristic of the whole model, not of specific entities. Example: Travel time between all station pairs, Number of parts in system, Simulation clock (built-in Arena variable)
  - Not tied to entities
- Resources
  - What entities compete for. Example: People, Equipment, Space
  - Entity obtains a resource, uses it, releases it
- Queues
  - Place for entities to wait
- Statistical accumulators
  - Variables that “watch” what’s happening
  - At end of simulation, used to compute final output performance measures

Randomness
- Probabilistic simulation technique used when a process has a random component
- A random variable (RV) is a number whose value is determined by the outcome of an experiment
- Probabilistic behavior of RV is described by distribution function
- Probability Mass Function (PMF)
  \[ p(x_i) = P(X = x_i) \] for \( i = 1, 2, ... \)
  - The statement “\( X = x_i \)” is an event that may or may not happen, so it has a probability of happening, as measured by the PMF
- Cumulative distribution function (CDF) = probability that the RV will be ≤ a fixed value x
Discrete Variables

- Data set has a "center" – the average (mean) \( \bar{X} \)
- RVs have a "center" – expected value
  \[ E(X) = \sum x_i p(x_i) \]
  - Also called the mean or expectation of the RV \( X \)
  - expectation is not: The value of \( X \) you "expect" to get
  \[ E(X) \] might not even be among the possible values \( x_1, x_2, \ldots \)
  \[ E(X) \] is what converges to (in a certain sense) as \( n \to \infty \)
- Data set has measures of "dispersion" –
  - Sample variance \[ s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 \]
  - Sample standard deviation \[ s = \sqrt{s^2} \]
- RV Variance \[ \text{Var}(X) = \sum \frac{1}{n} (x_i - \mu)^2 p(x_i) \]

Example

<table>
<thead>
<tr>
<th>Service Time (Min)</th>
<th>Probability</th>
<th>Cum. Prob</th>
<th>Random Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>0.20</td>
<td>RD: 48, 69, 97, 9, 4, 33</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
<td>0.30</td>
<td>11-30</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>0.60</td>
<td>31-60</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.85</td>
<td>61-85</td>
</tr>
<tr>
<td>5</td>
<td>0.10</td>
<td>0.95</td>
<td>86-95</td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>1.00</td>
<td>96-99</td>
</tr>
</tbody>
</table>

- What is the average service time?
  \[ 3 + 4 + 6 + 1 + 1 + 3 = 18/6 = 3 \text{ minutes} \]
- What is the expected service time?
  \[ 1(0.20) + 2(0.30) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05) = 3.2 \text{ minutes} \]

Continuous Variables

- Possibly limited to a range bounded on left or right or both
- No matter how small the range, the number of possible values for \( X \) is always infinite
- Expectation or mean of \( X \) is (average of a large number (infinite) of observations on the RV \( X \))
  \[ \mu = \mu_X = E(X) = \sum_{i=1}^{\infty} x_i f_i(x_i) dx \]
- Variance of \( X \) is
  \[ \sigma^2 = \sigma_X^2 = \text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx \]

Example

Life of a X-Ray device used to inspect cracks in aircraft wings is given by \( X \), a continuous random variable assuming all values in the range \( x \geq 0 \). The pdf of the life time, in years, is as follows:

\[ f(x) = \frac{1}{2} e^{-x/2} \quad x \geq 0 \]

What is the probability that the life of the X-Ray device is between 2 and 3 years?

\[ P(2 \leq X \leq 3) = \int_{2}^{3} f(x) dx = - e^{-3/2} + e^{-1} = - 0.223 + 0.368 = 0.145 \]

What is the expectation (mean) of the life of the X-Ray device?

\[ E(X) = \int_{-\infty}^{\infty} x f(x) dx = - xe^{-x/2} \bigg|_{0}^{\infty} = 2 \text{ years} \]

Independent RVs

- Properties of independent RVs:
  - They have nothing (linearly) to do with each other
  - Independence \( \Rightarrow \) uncorrelated
- Independence in simulation
  - Input: Usually assume separate inputs are independent
  - Output: Standard statistics assumes independence

Example

\[ \text{Cov}(X_1, X_2) = \frac{1}{n-1} \sum (x_i - \bar{x}_1)(x_i - \bar{x}_2) \]

\[ \text{Cor}(X_1, X_2) = \frac{\text{Cov}(X_1, X_2)}{\sigma_{X_1} \sigma_{X_2}} \]

- \( \text{Cor}(X_1, X_2) > 0 \) means +ve Correlation
- \( X_1 \) and \( X_2 \) move in the same direction \( \uparrow \uparrow \)
- \( \text{Cor}(X_1, X_2) = 0 \) means no correlation
- \( \text{Cor}(X_1, X_2) < 0 \) means –ve correlation \( X_1 \uparrow, X_2 \downarrow \)
Example

Let $X_1$ represent the average lead time to deliver (in months), and $X_2$ the annual demand, for industrial robots. The following data were available on demand and lead time for the last 10 years:

<table>
<thead>
<tr>
<th>Lead Time</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>103</td>
</tr>
<tr>
<td>4.3</td>
<td>83</td>
</tr>
<tr>
<td>6.9</td>
<td>161</td>
</tr>
<tr>
<td>6.0</td>
<td>97</td>
</tr>
<tr>
<td>6.9</td>
<td>112</td>
</tr>
<tr>
<td>6.9</td>
<td>104</td>
</tr>
<tr>
<td>3.8</td>
<td>186</td>
</tr>
<tr>
<td>7.3</td>
<td>109</td>
</tr>
<tr>
<td>4.5</td>
<td>92</td>
</tr>
<tr>
<td>6.3</td>
<td>96</td>
</tr>
</tbody>
</table>

$\sum X_1 = 6.14, \quad \sum X_2 = 101.80, \quad \sigma_1 = 1.02, \quad \sigma_2 = 9.93$

Covariance $= \frac{\sum X_1 X_2 - (\sum X_1)(\sum X_2)}{10-1} = \frac{6328.5 - (10)(6.14)(101.80)}{9} = 8.66$

Correlation $= \frac{8.66}{(1.02)(9.93)} = 0.86$

Cor (X1, X2) > 0 means +ve Correlation

Therefore lead time and demand or strongly dependent

Confidence Intervals

- A point estimator is just a single number, with some uncertainty or variability associated with it
- Confidence interval quantifies the likely imprecision in a point estimator
- An interval that contains (covers) the unknowns population parameter with specified (high) probability $1 - \alpha$
- Called a 100 $(1 - \alpha)\%$ confidence interval for the parameter
- Confidence interval for the population mean $\mu$:

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$t_{\alpha/2}$ is point below which is area $1 - \alpha/2$ in Student's $t$ distribution with $n-1$ degrees of freedom

Hypothesis Testing in Simulation

- Input side
  - Specify input distributions to drive the simulation
  - Collect real-world data on corresponding processes
  - “Fit” a probability distribution to the observed real-world data
  - Test $H_0$: the data are well represented by the fitted distribution
- Output side
  - Have two or more “competing” designs modeled
  - Test $H_0$: all designs perform the same on output, or test $H_0$: one design is better than another

Queuing Theory

- Queuing theory: Mathematical approach to the analysis of waiting lines.
- Goal of queuing analysis is to minimize the sum of two costs
  - Customer waiting costs
  - Service capacity costs

Queuing Model Notation

A / B / C / N / K

- A : Inter arrival time distribution
- B : Service time distribution
- C : Capacity or Number of parallel servers
- N : Queue size
- K : Size of population

Example: M / M / 1 / $\infty / \infty$

Means Expo arrival, expo service, 1 server, with $\infty$ queue size and $\infty$ calling population
Performance Measures

- \( L = E[\text{Expected average number in system}] = \sum_{n=0}^{\infty} n P_n \)
- \( \text{Utilization of servers} = p = \frac{\text{Average arrival rate}}{\text{Average service rate}} \)
- For \( G/G/1/\infty/\infty \) system, \( p = \frac{\lambda}{\mu} \)
- For \( G/G/c/\infty/\infty \) system, \( p = \frac{c \lambda}{\mu} \)
- The system is stable if and only if \( \lambda < cp \) (same as \( c > p \))

Example

Arrivals occur at rate \( \lambda = 10 \) per hour. Management has a choice to hire one of the two workers. One who works at rate \( \mu_1 = 11 \) customers and the second with \( \mu_2 = 12 \) customers per hour.

Utilization for worker 1: \( p_1 = \frac{10}{11} = 0.909 \),
Utilization for worker 2: \( p_2 = \frac{10}{12} = 0.833 \)

Average number in system for worker 1: \( \frac{\lambda}{\mu_1 - \lambda} = 10 \)
Average number in system for worker 2: \( \frac{\lambda}{\mu_2 - \lambda} = 5 \)

A decrease in service rate from 12 to 11 customers per hour (8.3%) would result in average number in system from 5 to 10 a 100% increase!

Queue Characteristics

- **Balk**
  - If the number of entities exceed the finite queue length, then entity goes back to calling population
  - Line too long or full
- **Renege**
  - Entities join a queue on arrival but later decides to leave (probably regretting not having balked in the first place!)
- **Jockey**
  - Line switching

Fitting Input Distributions

- Assume:
  - Have sample data: Independent and Identically Distributed (IID) list of observed values from the actual physical system
  - Want to select or fit a probability distribution for use in generating inputs for the simulation model
- **Arena Input Analyzer**
  - Separate application, also accessible via Tools menu in Arena
  - Fits distributions, gives valid Arena expression for generation to paste directly into simulation model
Fitting Distributions (Cont'd.)

- Fitting = deciding on distribution form (exponential, gamma, empirical, etc.) and estimating its parameters
  - Several different methods (Maximum likelihood, moment matching, least squares, ...)
  - Assess goodness of fit via hypothesis tests
    - H0: fitted distribution adequately represents the data
    - Chi square, Kolmogorov-Smirnov tests
    - Most important part: p-value, always between 0 and 1:
      - “Small” p (< 0.05 or so): poor fit (try again or give up)
  - Fitted “theoretical” vs. empirical distribution
  - Continuous vs. discrete data, distribution
  - “Best” fit from among several distributions (Fit/Fit All menu) or

Simulation Results Based on 100,000 Delays

<table>
<thead>
<tr>
<th>Service-time distribution</th>
<th>Average delay</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull*</td>
<td>4.36</td>
<td>-</td>
</tr>
<tr>
<td>exponential</td>
<td>6.71</td>
<td>53.9</td>
</tr>
<tr>
<td>normal</td>
<td>6.04</td>
<td>38.5</td>
</tr>
<tr>
<td>lognormal</td>
<td>7.19</td>
<td>64.9</td>
</tr>
</tbody>
</table>

*Best fit

Nonstationary Arrival Processes

- External events (often arrivals) whose rate varies over time
  - Lunchtime at fast-food restaurants
  - Rush-hour traffic in cities
  - Telephone call centers
  - Seasonal demands for a manufactured product
- It can be critical to model this nonstationarity for model validity
  - Ignoring peaks, valleys can mask important behavior
  - Can miss rush hours, etc.
- Good model: *Nonstationary Poisson process*

Nonstationary Arrival Processes (cont'd.)

- Two issues:
  - How to specify/estimate the rate function
  - How to generate from it properly during the simulation
- Several ways to estimate rate function — we'll just do the piecewise-constant method
  - Divide time frame of simulation into subintervals of time over which you think rate is fairly flat
  - Compute observed rate within each subinterval
  - Be careful about time units!

Output Analysis
Types of Statistics Reported

- Many output statistics are one of three types:
  - **Tally** – avg., max, min of a discrete list of numbers
    - Used for discrete-time output processes like waiting times in queue, total times in system
  - **Time-persistent** – time-average, max, min of a plot of something where the x-axis is continuous time
    - Used for continuous-time output processes like queue lengths, WIP, server-busy functions (for utilizations)
  - **Counter** – accumulated sums of something, usually just counts of how many times something happened
    - Often used to count entities passing through a point in the model

Time Frame of Simulations

- **Terminating** – Specific starting, stopping conditions
  - Run length will be well-defined (and finite; Known starting and stopping conditions)
- **Steady-state** – Long-run (technically forever)
  - Theoretically, initial conditions don’t matter (but practically they usually do)
  - Not clear how to terminate a simulation run (theoretically infinite)
  - Interested in system response over long period of time
- This is really a question of intent of the study
- Has major impact on how output analysis is done
- Sometimes it’s not clear which is appropriate

Half Width and Number of Replications

- Prefer smaller confidence intervals — precision
- Notation:
  - $n$ = no. replications
  - $X$ = sample mean
  - $s$ = sample standard deviation
  - $t_{n-1; .025}$ = critical value from t tables
- Confidence interval: 
  $$X \pm t_{.025} \frac{s}{\sqrt{n}}$$
- Half-width = $t_{.025} \frac{s}{\sqrt{n}}$ — how small?
- Can’t control $t$ or $s$
- Must increase $n$ — how much?

Half Width and Number of Replications (cont’d.)

- Set half-width $= h$ solve for $n$
  $$n = \frac{t_{.025}^2 \frac{s^2}{h^2}}{1}$$
- Not really solved for $n$ ($t$, $s$ depend on $n$)
- Approximation:
  - Replace $t$ by $z$, corresponding normal critical value
  - Pretend that current $s$ will hold for larger samples
  - Get $$n = \frac{z^2 s^2}{h^2}$$
- Easier but different approximation:
  $$n \approx n_0 \left( \frac{h_0}{h} \right)^2$$
  - $n_0$ = half width from “initial” number $n_0$ of replications
  - $h_0$ = half width from
  - $h$ decreases

Interpretation of Confidence Intervals

- Interval with random (data-dependent) endpoints that’s supposed to have stated probability of containing, or covering, the expected value
  - “Target” expected value is a fixed, but unknown, number
  - Expected value = average of infinite number of replications
- Not an interval that contains, say, 95% of the data
  - That’s a prediction interval … useful too, but different
- Usual formulas assume normally-distributed data
  - Never true in simulation
  - Might be approximately true if output is an average, rather than an extreme
  - Central limit theorem

Finding the Best System (Comparing Alternative Solutions)
Paired \( t \)-Test

- \( H_0: \mu_X - \mu_Y = 0 \)

- Take \( n \) observations from both strategies

- Set \( D_i = X_i - Y_i \) for \( i = 1, 2, \ldots, n \)

\[
\bar{D}_n = \frac{1}{n} \sum_{i=1}^{n} D_i
\]

\[
S_D^2 = \frac{1}{n-1} \sum_{i=1}^{n} (D_i - \bar{D}_n)^2
\]

- 100(1-\( \alpha \))% confidence interval:

\[
\bar{D}_n \pm t_{\alpha/2, n-1} \sqrt{\frac{S_D^2}{n}}
\]

Paired \( t \)-Test Cont...

- Find the CI

- IF CI includes \( 0 \), \( H_0 \) is true. (Improved system is not better)

\[
\mu_X - \mu_Y \in (-\infty, 0)
\]

Strong evidence that \( \mu_X < \mu_Y \)

System Y is better than X

\[
\mu_X - \mu_Y \in (0, \infty)
\]

Weak evidence that one system is better than the other

\[
\mu_X - \mu_Y \in (-\infty, \infty)
\]

Strong evidence that \( \mu_X \neq \mu_Y \)

- Use Welch approach if the number of replications are different

All Pair-wise Comparison

- Form simultaneous CI for \( \mu_i - \mu_j \) for all \( i \neq j \)

- Systems are simulated independently

- i.i.d. outputs \( Y_{i1}, \ldots, Y_{iR_i} \)

\[
\bar{Y}_i = \frac{1}{R_i} \sum_{j=1}^{R_i} Y_{ij}
\]

(Sample mean \( Y \)’s)

\[
S_i^2 = \frac{1}{R_i - 1} \sum_{j=1}^{R_i} (Y_{ij} - \bar{Y}_i)^2
\]

(Sample variance of \( Y \)’s)

- Tukey’s simultaneous confidence interval’s are:

\[
\bar{Y}_i - \bar{Y}_j \pm t_{\alpha, \nu} \sqrt{\frac{S_i^2}{R_i} + \frac{S_j^2}{R_j}}
\]

where \( \nu \approx \frac{\nu}{\nu + 1} \)

Coverage \( \geq 1 - \alpha \) for any values of the \( R_i \)

Comparison in Arena

- Compare Means via the Output Analyzer

  - Analyze > Compare Means menu option

  - Add data files … “A” and “B” for the two alternatives

  - Select “Lumped?” for Replications field

  - Title, confidence level, accept Paired-t Test, Scale Display

- PAN

  - Start PAN from Arena (Tools > Process Analyzer) or via Windows

- OptQuest

  - OptQuest searches intelligently for an optimum

  - Like PAN, OptQuest runs as a separate application … can be launched from Arena

Verification and Validation

- Incorrect data

- Mixed units of measure

  - Hours Vs. Minutes

- Blockages and deadlocks

  - Seize a resource but forgot to release

  - Forgot to dispose the entity at the end

- Incorrectly overwriting attributes and variables

  - Names

- Incorrect indexing

  - When you index beyond available queues and resources
V & V

This checking process consists of two main components:

**Verification:** Is “Code” = Model? (debugging)
- Determine if the computer implementation of the conceptual model is correct. Does the computer code represent the model that has been formulated?

**Validation:** Is Model = System?
- Determine if the conceptual model is a reasonable representation of the real-world system.

V & V is an iterative process to correct the “Code” errors and modify the conceptual model to better represent the real-world system.

The Truth: Can probably never completely verify, especially for large models.

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**Steady State Simulation**

**Techniques for Steady State Simulation**

- The main difficulty is to obtain independent simulation runs with exclusion of the transient period:
  - Waste of data of initial warm-up period
  - Inappropriate selection of warm-up period
- If model warms up very slowly, truncated replications can be costly
  - Have to “pay” warm-up on each replication
- Two techniques commonly used for steady state simulation are:
  - Method of Batch means, and
  - Independent Replication.
- None of these two methods is superior to the other in all cases.

**Warm Up and Run Length**

- Remedies for initialization bias
  - Better starting state, more typical of steady state
    - Throw some entities around the model
    - How do you know how many to throw and where?
      - This is what you’re trying to estimate in the first place!
    - Make the run so long that bias is overwhelmed
      - Might work if initial bias is weak or dissipates quickly
      - Let model warm up, still starting empty and idle
        - Run > Setup > Replication Parameters Warm-up Period
        - Time units!
        - “Clear” all statistics at that point for summary report, any Outputs saved data from Statistic module of results across replications

**Method of Independent Replications (cont’d.)**

- Suppose you have \( n \) equal batches of \( m \) observations each.

\[
\text{The mean of each batch is: } \text{mean}_i = \frac{\sum X_{ij}}{m}
\]

Overall estimate is: \( \text{Estimate} = \frac{\sum \text{mean}_i}{n} \)

The 100(1 - \( \alpha/2 \))% CI using \( t \) table is: \[ \text{Estimate} \pm t \cdot S \]

Where the variance \( S^2 = \frac{\sum (\text{mean}_i - \text{Estimate})^2}{n - 1} \)

**Batching in a Single Run**

- Just one long run
  - Only have to “pay” warm-up once
  - Problem: Have only one “replication” and you need more than that to form a variance estimate (the basic quantity needed for statistical analysis)

- Break each output record from the run into a few large batches
- Take averages over batches as “basic” statistics for estimation: Batch means
- Treat batch means as IID
  - Key: batch size must be big enough for low correlation between successive batches
**Batching in a Single Run (cont’d.)**

- Suppose you have \( n \) equal batches of \( m \) observations each.

The mean of each batch is: \( \text{mean}_i = \frac{\sum X_{ij}}{m} \)

Overall estimate is: \( \text{Estimate} = \frac{\sum \text{mean}_i}{n} \)

The \( 100(1 - \alpha/2)\% \) CI using \( t \) table is: \( \text{Estimate} \pm t \frac{S}{\sqrt{n}} \)

Where the variance \( S^2 = \frac{\sum (\text{mean}_i - \text{Estimate})^2}{n-1} \)

Wider CI means inaccurate estimation

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**Random Number Generation**

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**Properties of RNG**

- Numbers produced should appear to be distributed uniformly on \([0, 1]\) and should not exhibit any correlation with each other.
- Generator should be fast.
- Should be able to reproduce a given stream of random numbers:
  - For debugging, and
  - To use identical random numbers in simulating different systems in order to obtain a more precise comparison
- Should be a provision in the generator to easily reproduce separate “streams” of random numbers.

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**Linear Congruential Generators (LCGs)**

- The most common of several different methods
- Generate a sequence of integers \( Z_0, Z_1, Z_2, \ldots \) via the recursion
  \[
  Z_i = (a Z_{i-1} + c) \pmod{m}
  \]
  \( a, c, \) and \( m \) are carefully chosen constants
  - If \( c > 0 \): mixed LCGs and
  - If \( c = 0 \): multiplicative LCGs
- Specify a seed \( Z_0 \) to start off
- “\( \pmod{m} \)” means take the remainder of dividing by \( m \) as the next \( Z_i \)
- All the \( Z_i \)'s are between 0 and \( m-1 \)
- Return the \( i^{th} \) “random number” as \( U_i = Z_i / m \)

---

**Tests for Random Numbers**

- Need to test uniformity and independence
  - Frequency test
    - Kolmogorov–Smirnov (K-S) test or Chi-Square test to compare distribution of the set of numbers generated to a uniform distribution
  - Runs Test
    - Uses Chi-Square test to compare the runs above and below the mean by comparing actual values with expected values
  - Autocorrelation Test
    - Tests the correlation between numbers and compares the sample correlation with expected correlation of zero
  - Gap Test
    - Counts the number of digits that appear between repetitions of a particular digit and then uses K-S test to compare with the expected size of gaps

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**Inverse Transform Technique**

- Generate a random number \( U \sim \text{UNIF}(0, 1) \)
- Set \( U = F(X) \) (CDF) and solve for \( X = F^{-1}(U) \)
  - Solving analytically for \( X \) may or may not be simple (or possible)
  - Sometimes use numerical approximation to “solve”
Common Random Numbers (CRN)

- Applies when objective is to compare two (or more) alternative configurations or models
  - Interest is in difference(s) of performance measure(s) across alternatives
  - Example:
    - A. Base case (as is)
    - B. 3.5% increase in business (interarrival-time mean falls from 13 to 12.56 minutes)
- Get sharper comparison if you subject all alternatives to the same "conditions"
  - Then observed differences are due to model differences rather than random differences in the "conditions"
  - For both A and B runs, same:
    - The "same" parts arrive at the same times
    - Be assigned same attributes (job type)
    - Have the same process times at each step
- Then observed differences will be attributable to system differences, not random bounce
  - There isn't any random bounce
- One approach is to dedicate a stream of random numbers to each place in the model
- Another approach is to assign to each entity, immediately upon its arrival, attribute values for all possible processing times, branching decisions, etc.

Material Handling

Types of Material Handling Devices

- Material-handling devices
  - **Transporters**—fork lifts, trucks, carts, wheelchairs
    - Usually place limits on numbers, capabilities of transporters, initial location
    - Like a Resource, except moveable
  - **Conveyors**—Belts, hook lines, escalators
    - Usually limit space on conveyor, speed
    - Non-accumulating vs. accumulating
- Two types of Transporters
  - **Free-Path (unconstrained)**
    - Travel time depends only on velocity, distance
    - Ignore "traffic jams" and their resulting delays (can move around an obstruction)
    - Transport time = wait time + travel time
  - **Guided**
    - Move in fixed path, predefined network
    - AGVs, intersections, dead locks, etc.
    - Transport time = wait time + travel time + blocking time (at signals)

Experimental Design, Sensitivity Analysis, and Optimization

Goal

- We use paired t-test, etc… to compare alternate systems configurations.
  - We assume that various configurations are given
- Now we deal with a situation in which there is less structure in the goal of the simulation study.
  - We might want to find out which of possibly many parameters and structural assumptions have the greatest effect on a performance measure
  - Or, which set of parameters lead to optimal solution

Available Tools

- **Experimental Design**
  - Provides a way of deciding before the runs are made, which particular configurations to simulate
  - Help understand how factors interact with each other
- **Metamodeling**
  - a simpler mathematical function that approximates the relationship between the dependent and independent variables in the simulation model
  - Describes relationship between design variables and output response
    - Regression Analysis, Response Surface Methodology
- **Optimization and Heuristic models**
  - Tabu search, simulated annealing, genetic algorithms, perturbation analysis
Honda Case Study

Biological Manufacturing Systems

- Biological Manufacturing Systems (BMS) have functions that imitate those of a biological organism:
  - It has Self-Recognition
  - It has Self-Organization
  - It has Adaptability
  - It can Evolve and Learn

Requirements

- Achievement of function corresponding to multi models
- Manufacturing system which can efficiently adapt to change in total production volume
- Agility and flexibility for the fluctuation of rate of production volume
- Reduction of lead time from order to production

Honda Auto Body Example

Comparison of Facility Cost with line-wise and line-less Welding Process

Line-wise

Conventional Fixed Robot

Autonomous Movable Robot

Line-less

Conventional shuttle Conveyer

Autonomous AGV

Lineless Welding Floor Model

Simulation Time

AGV Queue

Dispatcher: 1-3

Collector: 1-3

Type-A

Type-B
Comparison examination with current line-wise model

<table>
<thead>
<tr>
<th>Comparison item</th>
<th>Line-wise model</th>
<th>Lineless(SOS)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Necessary robot number</td>
<td>88</td>
<td>58</td>
<td>A type + B type</td>
</tr>
<tr>
<td>Necessary AGV number</td>
<td>31</td>
<td>25</td>
<td>Include return AGVs</td>
</tr>
<tr>
<td>Floor Size</td>
<td>150mx25m</td>
<td>75mx50m</td>
<td>Equivalent</td>
</tr>
<tr>
<td>Productivity</td>
<td>1087</td>
<td>1140</td>
<td>daily (14.5 hr)</td>
</tr>
<tr>
<td>Robot Availability</td>
<td>41%</td>
<td>65%</td>
<td>Include Robot Invlxes</td>
</tr>
</tbody>
</table>

Summary of Benefits

- Robot availability will be higher than the line-wise system by 58% (including robot's malfunction)
- Robustness to demand change will be higher than the line-wise system
- Total operating cost per year will be lower than the line-wise system by 88%.
- Self-organization will reduce robot operations by 11% and process duration by 30%.